BIRLA INSTITUTE OF TECHNOLOGY

MESRA – 835215, RANCHI, INDIA

UG

Name: ………………………………………………………. Roll No.: …………………………………………..

Branch: ………………………………………………………. Signature of Invigilator: ……………………

Semester: VIth Date: 26/04/2022 (MORNING)

Subject with Code: **MA311 NUMERICAL TECHNIQUES**

Marks Obtained	Section A (30)	Section B (20)	Total Marks (50)

INSTRUCTION TO CANDIDATE

1. The booklet (question paper cum answer sheet) consists of two sections. First section consists of MCQs of 30 marks. Candidates may mark the correct answer in the space provided / may also write answers in the answer sheet provided. The Second section of question paper consists of subjective questions of 20 marks. The candidates may write the answers for these questions in the answer sheets provided with the question booklet.
2. The booklet will be distributed to the candidates before 05 minutes of the examination. **Candidates should write their roll no. in each page of the booklet.**
3. Place the Student ID card, Registration Slip and No Dues Clearance (if applicable) on your desk. All the entries on the cover page must be filled at the specified space.
4. Carrying or using of mobile phone / any electronic gadgets (except regular scientific calculator)/chits are strictly prohibited inside the examination hall as it comes under the category of unfair means.
5. No candidate should be allowed to enter the examination hall later than 10 minutes after the commencement of examination. Candidates are not allowed to go out of the examination hall/room during the first 30 minutes and last 10 minutes of the examination.
6. Write on both side of the leaf and use pens with same ink.
7. The medium of examination is English. Answer book written in language other than English is liable to be rejected.
8. All attached sheets such as graph papers, drawing sheets etc. should be properly folded to the size of the answer book and tagged with the answer book by the candidate at least 05 minutes before the end of examination.
9. The door of examination hall will be closed 10 minutes before the end of examination. Do not leave the examination hall until the invigilators instruct you to do so.
10. Always maintain the highest level of integrity. Remember you are a BITian.
11. Candidates need to **submit the question paper cum answer sheets before leaving the examination hall**.

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(**END SEMESTER EXAMINATION**)

Class: IMSC | **Semester**: VI
Branch: Maths & Computing | **Session**: SP/2022
Subject: MA311-Numerical Technique
Time: 2 Hours | **Full Marks**: 50
Date of Exam: 26/04/2022

Instructions:
1) The abbreviation T stands for TRUE and F stands for FALSE.
2) Paper contains two sections: (A) MCQs of 30 marks (B) Short Answer type of 20 marks.
3) Before attempting the question paper, be sure that you have got the correct question paper.
4) The missing data, If any, may be assumed suitably.

Section-A (MCQs) Attempt all questions.

1. The scheme $x_{n+1} = \frac{1}{2}x_n\left(1+\frac{\xi}{x_n^2}\right)$ converges to $\sqrt{\xi}$. The rate of convergence is [2 Marks]
(a)1 (b) 2 (c) 3 (d) 1.613

2. If $x = \xi$ is a double root of $f(x) = 0$ then the iteration scheme for determining ξ: $x_{n+1} = x_n - 2\frac{f(x_n)}{f'(x_n)}$ has the order [2 Marks]
(a) 2 (b) 1 (c) less than 2 (d)less than 1

3. If $f(x)$ has an isolated zero of multiplicity 3 at $x = \xi$ and the iteration $x_{n+1} = x_n - 3\frac{f(x_n)}{f'(x_n)}$ coverers to ξ, then the rate of convergence is [2 Marks]
(a) 1 (b) 1.5 (c) 2 (d) 3

4. The first approximation of the root lies in $(0, 1)$ of $x^3 + 3x - 1 = 0$ by Newton-Raphson is [2 Marks]
(a)0.3578 (b) 0.3588 (c) 0.3333 (d) 0.5333

5. Using Euclidean norm, the condition number of $A = \begin{bmatrix} 41 & 40 \\ 40 & 39 \end{bmatrix}$, is [2 Marks]
(a) 6402 (b) 6523 (c) 6645 (d) 6955

6. If A is a strictly diagonally matrix, then [2 Marks]
P. Jacobi iteration method converges for any initial guess.
Q. Gauss-Seidel iteration method converges for any initial guess.
(a) P is T, Q is T (b) P is F, Q is F
(c) P is T, Q is F (d) P is F, Q is T

7. The convergence rate of the scheme: $x^{n+1} = Hx^n + c$, is equal to [2 Marks]
(a) $r = -\ln[\rho(H)]$ (b) $r = -\log_{10}[\rho(H)]$
(c) Both (a) and (b) (d) Only (a)

8. The iteration scheme: $x^{n+1} = Hx^n + c$ for solving $Ax = b$ converges to the exact solution for any initial guess, if [2 Marks]
(a) $\|H\| < 1$ (b) $\|H\| \leq 1$ (c) $\|H\| \doteq 1$ (d) $\|H\| > 1$

9. The truncation error $E_1(x) = P_1(x) - f(x)$ for linear Lagrange method is [2 Marks]
(a) $(x - x_0)(x - x_1)\dfrac{f''(c)}{2!}$ (b) $(x - x_0)(x - x_1)\dfrac{f'''(c)}{3!}$
(c) $\dfrac{(x - x_0)f''(c)}{2}$ (d) None of these

10. If $f(x) = e^{10x}$, then $\Delta^n e^{10x}$ is equal to [2 Marks]
(a) $(e^{10h} - 1)^n e^{10x}$ (b) $(e^{10h} - 1)e^{10x}$
(c) $(e^{10h} + 1)^n e^{10x}$ (d) $5(e^{10h} - 1)^n e^{10x}$

11. The relation between E and Δ is [2 Marks]
(a) $E = 1 + \Delta$ (b) $E = 1 - \Delta$
(c) $E = \Delta - 1$ (d) $E = \dfrac{1}{1 + \Delta}$

12. The composite trapezoidal rule for $n = 10$, is [2 Marks]
(a) $\dfrac{h}{2}[y_0 + y_{10}] + h\displaystyle\sum_{i=1}^{9} y_i$ (b) $\dfrac{h}{2}\left[y_0 + 2\displaystyle\sum_{i=1}^{10} y_i + y_n\right]$
(c) $\dfrac{h}{2}\left[2\displaystyle\sum_{i=0}^{9} y_i + y_n\right]$ (d) $\dfrac{h}{2}\left[2\displaystyle\sum_{i=2}^{8} y_i + y_{10}\right]$

13. The numerical value of $\displaystyle\int_0^{\pi/2} (1 - 0.162\sin^2 x)^{1/2} dx$, using Simpson's 1/3 rule, taking 6 equal subintervals [2 Marks]
(a) 1.875 (b) 1.955 (c) 1.505 (d) 1.9999

14. The Euler-Method for solving $y' = f(x, y), y(x_0) = y_0$, is [2 Marks]
(a) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$ (b) $y_{n+1} = y_n + hf(x_n, y_n)$
(c) $y_{n+1} = y_n - hf(x_n, y_n)$ (d) None of these

15. The second order RK-Method for solving $y' = f(x, y), y(x_0) = y_0$, is [2 Marks]
(a) $y_{n+1} = y_n + \dfrac{h}{2}[f(x_n, y_n) - f(x_n + h, y_n + k_1)]$
(b) $y_{n+1} = y_n + \dfrac{1}{2}[f(x_n, y_n) - f(x_n + h, y_n + k_1)]$
(c) $y_{n+1} = y_n - \dfrac{1}{2}[f(x_n, y_n) + f(x_n + h, y_n + k_1)]$
(d) None of these

Section-B (Short Answer type Questions): Attempt any 5 questions.

1. Show that the convergence rate of the Newton-Raphson method is quadratic. [4 Marks]

2. Show that the convergence rate of the secant method is 1.62 method is quadratic. [4 Marks]

3. Perform three iteration of the power method to find the largest eigenvalue of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. [4 Marks]

4. Using the Newton divided difference interpolation, find the interpolation polynomial for data: $f(0) = 1, f(1) = 3, f(3) = 55$. [4 Marks]

5. Find the numerical value of $I = \int_{-1}^{1} x^2 e^{-x} dx$, using 3/8th Simpson's rules. [4 Marks]

6. Using fourth order R-K Method, evaluate $y(1.1)$ given that $y' + y/x = 1/x^2,\ y(1) = 1$ with $h = 0.1$. [4 Marks]