



Name: Roll No.:

Branch: Signature of Invigilator:

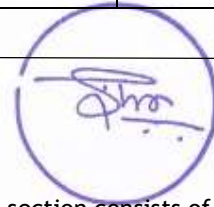
Semester: IVth

Date: 04/05/2022 (MORNING)

Subject with Code: MA306 SPECIAL FUNCTIONS

Marks Obtained	Section A (30)	Section B (20)	Total Marks (50)

INSTRUCTION TO CANDIDATE



1. The booklet (question paper cum answer sheet) consists of two sections. First section consists of MCQs of 30 marks. Candidates may mark the correct answer in the space provided / may also write answers in the answer sheet provided. The Second section of question paper consists of subjective questions of 20 marks. The candidates may write the answers for these questions in the answer sheets provided with the question booklet.
2. The booklet will be distributed to the candidates before 05 minutes of the examination. Candidates should write their roll no. in each page of the booklet.
3. Place the Student ID card, Registration Slip and No Dues Clearance (if applicable) on your desk. All the entries on the cover page must be filled at the specified space.
4. Carrying or using of mobile phone / any electronic gadgets (except regular scientific calculator)/chits are strictly prohibited inside the examination hall as it comes under the category of unfair means.
5. No candidate should be allowed to enter the examination hall later than 10 minutes after the commencement of examination. Candidates are not allowed to go out of the examination hall/room during the first 30 minutes and last 10 minutes of the examination.
6. Write on both side of the leaf and use pens with same ink.
7. The medium of examination is English. Answer book written in language other than English is liable to be rejected.
8. All attached sheets such as graph papers, drawing sheets etc. should be properly folded to the size of the answer book and tagged with the answer book by the candidate at least 05 minutes before the end of examination.
9. The door of examination hall will be closed 10 minutes before the end of examination. Do not leave the examination hall until the invigilators instruct you to do so.
10. Always maintain the highest level of integrity. Remember you are a BITian.
11. Candidates need to submit the question paper cum answer sheets before leaving the examination hall.

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI, INDIA

END SEMESTER EXAMINATION SP/2022

CLASS- BTECH

SEMESTER- IV

SUBJECT- MA306, SPECIAL FUNCTIONS

TIME-2 HOURS; MM-50

INSTRUCTIONS:

- I. Question paper contains two sections A and B containing in all 19 questions. In section A, Q1-Q15 is MCQ type of two marks each. In section B, Q16-Q19 is short answer type of 5 marks each. Attempt all questions. All questions are compulsory.
- II. The missing data if any, may be assumed suitably. The symbols used have their usual meaning.
- III. There is no negative marking.

Section A

1. Lagrange polynomial of degree n satisfies
 - a. $P_n(x) = F(-n, n+1; 1; (1-x)/2)$
 - b. $P_n(x) = F(-n, -(n+1); 1; (1-x)/2)$
 - c. $P_n(x) = F(n, n+1; 1; (1-x)/2)$
 - d. None of these
2. The Hermite polynomial of order $2n$ satisfies
 - a. $H_{2n}(x) = (-1)^n \frac{(2n)!}{n!} F(-n; 1/2; x^2)$
 - b. $H_{2n}(x) = (-1)^{n+1} \frac{(2n)!}{n!} F(-n; 1/2; x^2)$
 - c. $H_{2n}(x) = (-1)^n \frac{(2n)!}{n!} F(n; 1/2; x^2)$
 - d. None of these
3. Chebyshev polynomial $T_n(x)$ of second kind satisfies
 - a. $T_n(x) = F(-n, n; 1/2; (1-x)/2)$
 - b. $T_n(x) = F(n, -n; 1/2; (1-x)/2)$
 - c. $T_n(x) = F(n, n; 1/2; (1-x)/2)$
 - d. None of these
4. Consider the following statements:
P: $e^x = F(\alpha; \alpha; x)$
Q: $\ln(1-x) = -xF(1, 1; 2; x)$
 - a. P is true Q is false
 - b. P is false, Q is true
 - c. Both P and Q are true
 - d. Both P and Q are false

5. Consider the following statements:

P: $T_n(x) - 2xT_{n-1}(x) + T_{n-2}(x) = 0$

Q: $L_n''(0) = \frac{1}{4}n(n-1)$

- a. P is true Q is false
- b. P is false, Q is true
- c. Both P and Q are true
- d. Both P and Q are false

6. Consider the following statements:

P: The value of $\int_{-\infty}^{\infty} e^{-x^2} (H_4(x))^2 dx$ is $8\sqrt{\pi}$

Q: The solution of the differential equation $y'' - 2xy' + 36y = 0$ is $H_{18}(x)$.

- a. P is true Q is false
- b. P is false, Q is true
- c. Both P and Q are true
- d. Both P and Q are false

7. Consider the following statements:

P: $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$

Q: $xL_n(x) = nL_n(x) - nL_{n-1}(x)$

- a. P is true Q is false
- b. P is false, Q is true
- c. Both P and Q are true
- d. Both P and Q are false

8. Consider the following statements:

P: $F(\alpha, \beta; \gamma; 1) = \frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}$

Q: $\int_0^{\infty} e^{-x} L_2(x)L_3(x)dx = 1$

- a. P is true Q is false
- b. P is false, Q is true
- c. Both P and Q are true
- d. Both P and Q are false

9. Consider the following statements:

P: $H_n'(x) = 2nH_{n-1}(x), n \geq 1$

Q: $H_{n+1}(x) = 2xH_n(x) + 2nH_{n-1}(x), n \geq 1$

- a. P is true Q is false
- b. P is false, Q is true
- c. Both P and Q are true
- d. Both P and Q are false

10. Consider the following statements:

P: $H_6(x) = 64x^6 - 480x^4 + 720x^2 - 120$

Q: General solution of $y'' - 2xy' + 16y = 0$ is $H_8(x)$

- a. P is true Q is false
- b. P is false, Q is true
- c. Both P and Q are true
- d. Both P and Q are false

11. Consider the following statements:

P: Solution of $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ is $1 - \frac{4}{3}x$

Q: $\int_0^\infty L_6^2(x)dx = 1$

- a. P is true Q is false
- b. P is false, Q is true
- c. Both P and Q are true
- d. Both P and Q are false

12. For the differential equation $x^2(1+x)y'' + x(4-x^2)y' + (2+3x)y = 0$, the point $x = 0$ is

- a. A regular singular point
- b. An irregular singular point
- c. Ordinary point
- d. None of these

13. For the differential equation $x^4y'' + (x^2 \sin x)y' + (1 - \cos x)y = 0$, the point $x = 0$ is

- a. Not a regular singular point
- b. Ordinary point
- c. A regular singular point
- d. None of these

14. The Fourier series of the 2π periodic function $f(t) = t, -\pi < t \leq \pi$ is

- a. $\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \dots\right)$
- b. $2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \dots\right)$
- c. $\pi - 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \dots\right)$
- d. None of these

15. The Fourier series of $f(t) = |t|, -\pi < t \leq \pi$ is

- a. $\frac{\pi}{2} - \frac{4}{\pi}\left(\cos t + \frac{1}{9}\cos 3t + \frac{1}{25}\cos 5t + \dots\right)$
- b. $\frac{\pi}{2} + \frac{4}{\pi}\left(\cos t + \frac{1}{9}\cos 3t + \frac{1}{25}\cos 5t + \dots\right)$
- c. $\frac{4}{\pi}\left(\cos t + \frac{1}{9}\cos 3t + \frac{1}{25}\cos 5t + \dots\right)$
- d. None of these

Section B

16. Find all eigen values and eigen functions of the Sturm-Liouville problem

$$X'' + \gamma X = 0, X'(-\pi) = 0, X'(\pi) = 0.$$

17. Show that $\int_{-\infty}^{\infty} x^2 e^{-x^2} (H_n(x))^2 dx = \sqrt{\pi} 2^n n! (n + 1/2)$

18. Show that $L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$

19. Find the Fourier series of $f(t) = |t|, -1 < t < 1$