BIRLA INSTITUTE OF TECHNOLOGY
MESRA – 835215, RANCHI, INDIA

UG

Name: ………………………………………………… Roll No.: ………………………………………..

Branch: ………………………………………………… Signature of Invigilator: ……………………

Semester: IVth Date: 29/04/2022 (MORNING)

Subject with Code: **MA209 INTEGRAL EQUATIONS AND GREENS FUNCTION**

Marks Obtained	Section A (30)	Section B (20)	Total Marks (50)

INSTRUCTION TO CANDIDATE

1. The booklet (question paper cum answer sheet) consists of two sections. First section consists of MCQs of 30 marks. Candidates may mark the correct answer in the space provided / may also write answers in the answer sheet provided. The Second section of question paper consists of subjective questions of 20 marks. The candidates may write the answers for these questions in the answer sheets provided with the question booklet.
2. The booklet will be distributed to the candidates before 05 minutes of the examination. **Candidates should write their roll no. in each page of the booklet.**
3. Place the Student ID card, Registration Slip and No Dues Clearance (if applicable) on your desk. All the entries on the cover page must be filled at the specified space.
4. Carrying or using of mobile phone / any electronic gadgets (except regular scientific calculator)/chits are strictly prohibited inside the examination hall as it comes under the category of unfair means.
5. No candidate should be allowed to enter the examination hall later than 10 minutes after the commencement of examination. Candidates are not allowed to go out of the examination hall/room during the first 30 minutes and last 10 minutes of the examination.
6. Write on both side of the leaf and use pens with same ink.
7. The medium of examination is English. Answer book written in language other than English is liable to be rejected.
8. All attached sheets such as graph papers, drawing sheets etc. should be properly folded to the size of the answer book and tagged with the answer book by the candidate at least 05 minutes before the end of examination.
9. The door of examination hall will be closed 10 minutes before the end of examination. Do not leave the examination hall until the invigilators instruct you to do so.
10. Always maintain the highest level of integrity. Remember you are a BITian.
11. Candidates need to **submit the question paper cum answer sheets before leaving the examination hall**.

BIRLA INSTITUTE OF TECHNOLOGY MESRA, RANCHI
(End Semester Examination)

Class: IMSC — Semester: IV
Branch: Mathematics and Computing — Session: SP-2022

Subject: MA209 Integral Equations and Green's Function

Time-2 hours — Full marks-50

Section A: Answer all [2X15=30]

1. The integral equation: $\int_0^x \frac{1}{\sqrt{x-t}} y(t)dt = \sqrt{x}$ has the solution: [2]

   (a) $y(x) = \frac{x}{2}$ (b) $y(x) = \frac{1}{2}$ (c) $y(x) = \frac{x^2}{2}$ (d) $y(x) = \frac{x^2}{2!}$

2. The initial value problem: $y''(x) + xy(x) = 1, \quad y'(0) = 0, \quad y(0) = 0$ is equivalent to the integral equation: [2]

   (a) $y(x) = \frac{x}{2} + \int_0^x x(x-t)\ y(t)\ dt$ (b) $y(x) = \frac{x}{2} + \int_0^x t(t-x)\ y(t)\ dt$
   (c) $y(x) = \frac{x^2}{2} + \int_0^x x(x-t)\ y(t)\ dt$ (d) $y(x) = \frac{x^2}{2} + \int_0^x t(t-x)\ y(t)\ dt$

3. The boundary value problem: $y''(x) + y(x) = 0,\ y(0) = 1,\ y'(1) = 0$ is equivalent to the integral equation: [2]

   (a) $y(x) = x + \int_0^1 k(x,t)\ y(t)\ dt$, where, $k(x,t) = \begin{cases} t, & x < t \\ x, & x > t \end{cases}$
   (b) $y(x) = 1 + \int_0^1 k(x,t)\ y(t)\ dt$, where, $k(x,t) = \begin{cases} t, & x < t \\ x, & x > t \end{cases}$
   (c) $y(x) = 1 + \int_0^1 k(x,t)\ y(t)\ dt$, where, $k(x,t) = \begin{cases} t, & t < x \\ x, & t > x \end{cases}$
   (d) none of these

4. The initial value problem corresponding to the integral equation: [2]
   $y(x) = 1 + \int_0^x y(t)\ dt$ is

   (a) $y'(x) - y(x) = 0, \quad y(0) = 1$ (b) $y'(x) - y(x) = 0, \quad y(0) = 0$
   (c) $y'(x) + y(x) = 0, \quad y(0) = 1$ (d) $y'(x) + y(x) = 0, \quad y(0) = 0$

5. The integral equation: $y(x) = \lambda \int_0^1 \sin(\pi x)\cos(\pi t)\ y(t)dt$ has [2]

   (a) atleast one eigen value and corresponding eigen function
   (b) infinitely many eigen values and corresponding eigen functions
   (c) no eigen value and eigen function
   (d) none of these

6. The eigen value of the integral equation: $y(x) = \lambda \int_0^{\pi/4} \sin^2 x \; y(t) \; dt$ is [2]

(a) $\lambda = \dfrac{8}{\pi - 2}$ (b) $\lambda = \dfrac{8}{\pi}$ (c) $\dfrac{4}{\pi - 2}$ (d) $\dfrac{4}{\pi}$

7. The solution of the integral equation: $y(x) = x + \int_0^1 xt^2 \; y(t) \; dt$ is given by: [2]

(a) $y(x) = \dfrac{3x}{4}$ (b) $y(x) = \dfrac{4x}{3}$ (c) $y(x) = \dfrac{2x}{3}$ (d) $y(x) = \dfrac{3x}{2}$

8. For the integral equation: $y(x) = f(x) + \lambda \int_a^x k(x,t) \; y(t) \; dt$, the iterated kernels $k_m(x,t)$ are defined by: [2]

(a) $k(x,t) = k_1(x,t)$; $k_m(x,t) = \int_a^x k(x,z) \; k_{m-1}(z,t) dz$

(b) $k(x,t) = k_1(x,t)$; $k_m(x,t) = \int_t^x k(x,z) \; k_{m-1}(z,t) dz$

(c) $k(x,t) = k_1(x,t)$; $k_m(x,t) = \int_t^x k_1(x,z) \; k_{m-1}(z,t) dt$

(d) $k(x,t) = k_1(x,t)$; $k_m(x,t) = \int_a^x k_1(x,z) \; k_{m-1}(z,t) dt$

9. The iterated kernel $K_2(x,t)$ for the integral equation $y(x) = f(x) + \lambda \int_0^{\pi} e^x \cos t \; y(t) dt$ is [2]

(a) $k_2(x,t) = -\dfrac{1+e^{\pi}}{2} e^x \cos t$, (b) $k_2(x,t) = \dfrac{1+e^{\pi}}{2} e^x \cos t$,

(c) $k_2(x,t) = -\dfrac{1-e^{\pi}}{2} e^x \cos t$, (d) $k_2(x,t) = \dfrac{1-e^{\pi}}{2} e^x \cos t$.

10. The iterated kernel $K_2(x,t)$ for the integral equation $y(x) = f(x) + \lambda \int_a^x \dfrac{2+\cos x}{2+\cos t} \; y(t) dt$ is [2]

(a) $k_2(x,t) = \dfrac{2+cosx}{2+cost}(x-a)$ (b) $k_2(x,t) = \dfrac{2+cosx}{2+cost}(x-t)$

(c) $k_2(x,t) = \dfrac{2+cosx}{2+cost}(a-x)$ (d) $k_2(x,t) = \dfrac{2+cosx}{2+cost}(t-x)$

11. The second order approximation of the integral equation: $y(x) = 1 + \lambda \int_0^1 (x+t) y(t) \; dt$ with $y_0(x) = 1$ is given by: [2]

(a) $y_2(x) = 1 + \lambda\left(x - \frac{1}{2}\right) + \lambda^2\left(x + \frac{7}{12}\right)$ (b) $y_2(x) = 1 + \lambda\left(x - \frac{1}{2}\right) + \lambda^2\left(x - \frac{7}{12}\right)$

(c) $y_2(x) = 1 + \lambda\left(x + \frac{1}{2}\right) + \lambda^2\left(x + \frac{7}{12}\right)$ (d) $y_2(x) = 1 + \lambda\left(x + \frac{1}{2}\right) + \lambda^2\left(x + \frac{7}{2}\right)$

12. A nonnull, symmetric $\mathcal{L}_2$ kernel $K(x,t)$ is non-negative if and only if: [2]
(a) all its eigenvalues are negative
(b) all its eigenvalues are positive
(c) some of its eigenvalues are negative
(d) none of these

13. The necessary and sufficient condition that the 2nd order homogeneous linear differential equation: $a_0(x)\dfrac{d^2y}{dx^2} + a_1(x)\dfrac{dy}{dx} + a_2(x)y = 0, \quad a \le x \le b,$ to be self adjoint is that: [2]

(a) $a_0(x) = a_1'(x)$ (b) $a_0'(x) = -a_1(x)$ (c) $a_1'(x) = -a_0(x)$ (d) $a_0'(x) = a_1(x)$

14. The euquivalent self-adjoint equation the differential equation $y'' - (\tan x)y' + y = 0$ is : [2]

(a) $(\cos x)y'' - (\sin x)y' + (\cos x)y = 0$ (b) $(\cos x)y'' + (\sin x)y' + (\cos x)y = 0$
(c) $(\sin x)y'' - (\cos x)y' + (\sin x)y = 0$ (d) none of these

15. The adjoint equation of: $x^2y'' + 7xy' + 8y = 0$ is [2]

(a) $x^2y'' - 3xy' + 3y = 0$ (b) $x^2y'' + 3xy' + 3y = 0$
(c) $x^2y'' - 3xy' - 3y = 0$ (d) $x^2y'' - 3xy' + y = 0$

Section B: Answer five question only: [4X5=20]

16. Prove that $\displaystyle\int_a^x \int_a^x \int_a^x ..... \int_a^x f(t)dt.........dtdtdt = \frac{1}{(n-1)!}\int_a^x (x-t)^{n-1} f(t)dt$. [4]

17. Prove that the resolvent kernel $R(x,t;\lambda)$ for the integral equation $y(x) = f(x) + \lambda \displaystyle\int_a^b k(x,t)y(t)dt$ satisfies the integral equation: $R(x,t;\lambda) = k(x,t) + \lambda \displaystyle\int_a^b k(x,z)R(z,t;\lambda)dz.$ [4]

or,

Prove that the resolvent kernel $R(x,t;\lambda)$ for the integral equation $y(x) = f(x) + \lambda \displaystyle\int_a^x k(x,t)y(t)dt$ satisfies the integral equation: $R(x,t;\lambda) = k(x,t) + \lambda \displaystyle\int_t^x k(x,z)R(z,t;\lambda)dz.$ [4]

18. Prove that if a kernel is symmetric then all its iterated kernels are symmetric. [4]

or,

Prove that the eigenfunctions of a symmetric kernel, corresponding to different eigenvalues are orthogonal. [4]

19. Using Green's function solve the following boundary value problems: [4]
$y''(x) + y(x) = x^2, \quad y(0) = 0, \quad y(\pi/2) = 0,$

20. Solve the following integro-differential equation: $y'(x) = x + \displaystyle\int_0^x \cos t y(x-t)dt, \quad y(0) = 4$ [4]

(********** 29-04-2022 **********)