BIRLA INSTITUTE OF TECHNOLOGY
MESRA – 835215, RANCHI, INDIA

UG

Name: …… Roll No.: ……..

Branch: …… Signature of Invigilator: ……

Semester: IVth Date: 27/04/2022 (MORNING)

Subject with Code: **MA205 DISCRETE MATHEMATICS**

	Section A (30)	Section B (20)	Total Marks (50)
Marks Obtained			

INSTRUCTION TO CANDIDATE

1. The booklet (question paper cum answer sheet) consists of two sections. First section consists of MCQs of 30 marks. Candidates may mark the correct answer in the space provided / may also write answers in the answer sheet provided. The Second section of question paper consists of subjective questions of 20 marks. The candidates may write the answers for these questions in the answer sheets provided with the question booklet.
2. The booklet will be distributed to the candidates before 05 minutes of the examination. **Candidates should write their roll no. in each page of the booklet.**
3. Place the Student ID card, Registration Slip and No Dues Clearance (if applicable) on your desk. All the entries on the cover page must be filled at the specified space.
4. Carrying or using of mobile phone / any electronic gadgets (except regular scientific calculator)/chits are strictly prohibited inside the examination hall as it comes under the category of unfair means.
5. No candidate should be allowed to enter the examination hall later than 10 minutes after the commencement of examination. Candidates are not allowed to go out of the examination hall/room during the first 30 minutes and last 10 minutes of the examination.
6. Write on both side of the leaf and use pens with same ink.
7. The medium of examination is English. Answer book written in language other than English is liable to be rejected.
8. All attached sheets such as graph papers, drawing sheets etc. should be properly folded to the size of the answer book and tagged with the answer book by the candidate at least 05 minutes before the end of examination.
9. The door of examination hall will be closed 10 minutes before the end of examination. Do not leave the examination hall until the invigilators instruct you to do so.
10. Always maintain the highest level of integrity. Remember you are a BITian.
11. Candidates need to **submit the question paper cum answer sheets before leaving the examination hall**.

BIRLA INSTITUTE OF TECHNOLOGY MESRA, RANCHI
DEPARTMENT OF MATHEMATICS
END SEMESTER EXAMIMATION

Course: MA205 Discrete Mathematics **Session: SP/2022**

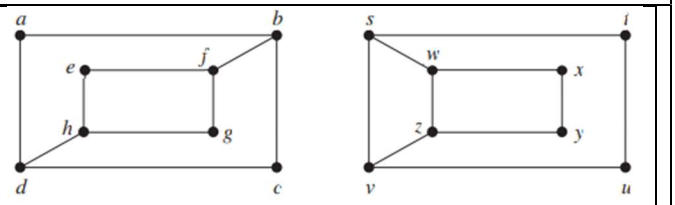
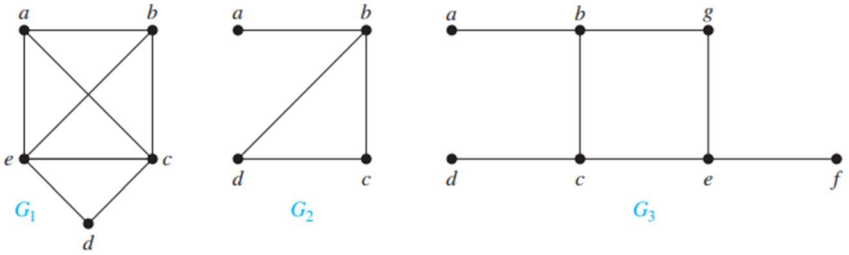
Time : 2 hours **Maximum Marks : 50**

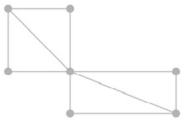
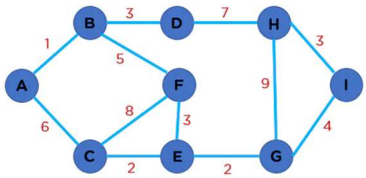
Section A

Each Question carry one mark

1	Identify the correct negation of two statements (i) $\forall x\ ,\ |x| = x$ (ii) $\exists x\ ,\ x^2 = x$ $(a)\ \forall x, |x| \neq x\ \ and\ \exists x, x^2 \neq x$ (b) $\forall x, |x| \neq x\ \ and\ \forall x, x^2 \neq x$ (c) $\forall x, |x| \neq x\ \ and\ \exists x, \sim(x^2 = x)$ (d) $\forall x, |x| \neq x\ \ and\ \forall x, \sim(x^2 = x)$
2	Identify the correct solution of recurrence relation $a_n + a_{n-1} = 3n \times 2^n$, (c given constant) a. $c(-1)^n + \left(2n + \frac{2}{3}\right) \times 2^n$ (b) $c(-1)^n + \left(2n + \frac{2}{3}\right)$ c. $c(-1)^n + (2n + 2) \times 2^n$ (d) $c(-1)^n + \left(2n + \frac{1}{3}\right) \times 2^n$ Identify the correct answer: (i)a→T, b→ T , c→T, d→ T (ii) a→T, b→ F, c→T, d→ T (iii) a→F, b→ T, c→T, d→ T (iv)a→F, b→ F, c→F, d→ F
3	Given P_1: $(p \wedge\sim q) \vee (q \wedge\sim p)$ & P_2: $(p \Rightarrow q) \vee (q \Rightarrow p)$, Identify the correct ones? $(i)P_1$ is a tautology but not P_2 (ii) P_2 is a tautology but not P_1 $(iii)P_1$ and P_2 both are tautologies (iv) P_1 and P_2 both are not tautologies (v)None
4	Let $Q_1 = 1$, $Q_2 = 2$ and $Q_3 = 3$, $Q_n = (n^3-3n^2+2n)\ Q_{n-3}$ for all n≥4. We prove that for every n belonging to N, $Q_n = n!$ (i.e factorial n). What cases will be needed to prove it correctly by induction. (a) n=2,3 (b) n=1,2,3 (c) n= 1,2,3,4 (d) n=1,2,3,4,5
5	a: For all n∈N, 6^{n-1} is a multiple of 5 using induction/strong induction. b: G be abelian iff $(ab)^2=a^2 b^2$, then (G,*) is abelian group. Identify the correct one from below (i)a→T, b→ T (ii)a→T, b→ F (iii) a→F, b→ T (iv) a→F, b→ F
6	Given the premises: C∨ D, C∨ D → ¬ H, ¬ H → (A ∧ ¬ B), (A ∧ ¬ B) → R ∨ S Does R ∨ S follow logically from above given premises? (i) No (ii) Yes (iii) May be (iv) Incorrect problem
7	The non-linear recurrence relation among below is/are (a) $a_n^2 - 2\ a_{n-1}^2$=1 for n ≥ 1 (b)$a_n = 5a_{n-1} - 6\ a_{n-2} + 7^n$ $(c)a_n = 5a_{n-1} - 3\ a_{n-2}$ $(d)a_n - 15a_{n-3} - 36\ a_{n-4} = 2^n$ (i)a (ii)a, b (iii) a, b, c (iv) a. b,c,d
8	What is/are the correct generating function of the sequence 0^2, 1^2, 2^2, 3^2 …. . (i) 1+z (b) 1/ [(1-z)(1-z)$^{-2}$] (c) Z(1+z)(1-z)$^{-3}$ (d) 1/{ Z(1+z) (1-z)$^{-3}$} (e) None
9	Let H be a subgroup of a group G. Then the relation $R = \{(x, y): x, y \in G, x^{-1}y \in H\}$ is an (i) Equivalence relation (ii) Is not equivalence relation (iii) Is Partial order relation (iv) None.
10	(a) The product of two odd permutations is even. (b) The product of one even and one odd permutations is even. Identify the correct one from below (i)a→T, b→ T (ii)a→T, b→ F (iii) a→F, b→ T, (iv)a→F, b→ F

11	(a) The semigroup (Z+, +) has identity element. (b) Let T be the set of all even integers. Then semigroups (Z, +) and (T, +) are not isomorphic. Identify the correct one from below (i)a→T, b→ T (ii)a→T, b→ F (iii) a→F, b→ T (iv)a→F, b→ F
12	(a) G = { 1, -1, i, - i, x), the set of all fourth root of unity forms a group. (b) If ° is an operation on Z defined by x* y=x+y+1, Prove that (Z,*) is a non abelian group. Identify the correct one from below (i)a→T, b→ T (ii)a→T, b→ F (iii) a→F, b→ T (iv)a→F, b→ F
13	(a) Every cyclic group is abelian, but converse is not true. (b) The group (Z_5*, •)is a cyclic group Identify the correct one from below (i)a→T, b→ T (ii)a→T, b→ F (iii) a→F, b→ T (iv)a→F, b→ F
14	(a) Consider the semigroup (Z, +) and the equivalence relation R on Z defined by aRb, if and only if a = b (mod 2). Then that this relation is not a congruence relation. (b) The weight of each of the following words in B^5: (a) x = 01000, |x|=1 (b) x = 11100 , |x|=2, (c) x = 00000, |x|=0 (d) x = 11111, |x|=5 Identify the correct one from below (i)a→T, b→ T (ii)a→T, b→ F (iii) a→F, b→ T (iv)a→F, b→ F
15	(a) The truth value of $\forall x\ P(x)$ is False, where $P(x)$: $x^2<10$ and universe of discourse consist of a positive integer not exceeding 4. (b) If P(x) denote the statement, Then $\exists x \in N$, (x+6<8) is true. Identify the correct one from below (i)a→T, b→ T (ii)a→T, b→ F (iii) a→F, b→ T (iv)a→F, b→ F
16	Arrange in following theta functions in ascending order (A) Θ(nlgn) B. $\Theta(1000n^2 - n)$ C. $\Theta(n^{0.2})$ D. Θ(1,000,000) E. $\Theta(1.3^n)$ F. $\Theta(n + 10^7)$ Identify the correct one (i) DEBAFC (ii) DBEAFC (iii) DEABFC (iv) DEBACF (v) DEBAFC (vi) None
17	Determine the value of a_2 for the recurrence relation $a_n = 17a_{n-1} + 30n$ with $a_0=3$. a) 4387 b) 5484 c) 238 d) 1437
18	What is the correct addition of two numeric functions form the options given below, Let $a_r = \begin{Bmatrix} 0, & 0 \le r \le 2 \\ 2^{-r}+5, & r \ge 3 \end{Bmatrix}$ and $b_r = \begin{Bmatrix} 3-2^r, & 0 \le r \le 1 \\ r+2, & r \ge 2 \end{Bmatrix}$ *Then Sum* c_r =? (i) $\begin{Bmatrix} 3-2^r, & 0 \le r \le 1 \\ 3, & r=2 \\ 2^{-r}+r+7, & r \ge 3 \end{Bmatrix}$ (ii) $\begin{Bmatrix} 3-2^r, & 0 \le r \le 1 \\ 4, & r=2 \\ 2^{-r}+r, & r \ge 3 \end{Bmatrix}$ (iii) $\begin{Bmatrix} 3-2^r, & 0 \le r \le 1 \\ 4, & r=2 \\ 2^{-r}+5, & r \ge 3 \end{Bmatrix}$ (iv) $\begin{Bmatrix} 3-2^r, & 0 \le r \le 1 \\ 4, & r=2 \\ 2^{-r}+r+7, & r \ge 3 \end{Bmatrix}$
19	Let $R = \{(x,y): x = y^2\}$ be a relation then (i) R is a function (ii) R is not a function (iii) R is one-one onto function (iv) None of these
20	Let *R* be a relation on a set *A*, where *A*={1,2,3,4,5}, *R*={(1,2),(2,3),(3,4),(4,5)}. What is the correct transitive closure *R* of *R* ?(Using any known method)

	(a) (1,2),(2,3),(3,4),(4,5), (1,3),(1,4),(1,5), (2,4), (3,5) (b) (1,2),(2,3),(3,4),(4,5), (1,4),(1,5), (2,4),(2,5), (3,5) (c) (1,2),(3,4),(4,5), (1,3),(1,4),(1,5), (2,4),(2,5), (3,5) (d) (1,2),(2,3),(3,4),(4,5), (1,3),(1,4),(1,5), (2,4),(2,5), (3,5)
21	If the set Z of integers is a group defined by $m * n = m + n + 1, m, n \in Z$ then inverse of element of 5 is (i) -9 (ii) 10 (iii) -7 (iv) -5
22	If $M_2(R)$ be the group of 2x2 real setoff matrix under matrix addition and define a trace map $tr: M_2(R) \to R$then trace map is (i) Isomorphism (ii) Homomorphism(iii) Non-Homomorphism (iv) None of these
23	If A= (1,2,3). Using Warshall's Algorithm, the transitive closure of relation R = $\begin{bmatrix}0&1&0\\0&1&1\\0&0&1\end{bmatrix}$ is/are (a) $\begin{bmatrix}0&1&0\\0&1&1\\1&0&1\end{bmatrix}$ (b) $\begin{bmatrix}0&1&1\\0&1&1\\0&0&1\end{bmatrix}$ (c) $\begin{bmatrix}0&1&0\\1&1&1\\0&0&1\end{bmatrix}$ (d) $\begin{bmatrix}1&1&0\\0&1&1\\0&0&1\end{bmatrix}$
24	Determine whether the graphs shown in Figure are isomorphic? (i) YES (ii) NO (ii) Can't find , More data is required 
25	Which of the statement is/are true about the simple graphs given below? (i) G1 has a Hamilton circuit, G2 has a Hamilton circuit, G3 has a Hamilton circuit (ii) G1 has a Hamilton circuit, G2 has a no Hamilton circuit, G3 neither a Hamilton circuit nor a Hamilton path (iii) G1 has does not have Hamilton circuit, G2 does not have a Hamilton circuit, G3 has a Hamilton circuit 
26	(a) If a graph G has a vertex of odd degree, there can be no Euler circuit in G. (b) If G is a connected graph and every vertex has even degree, then there is an Euler circuit in G. Identify the correct one from below (i)a→T, b→ T (ii)a→T, b→ F (iii) a→F, b→ T, (iv)a→F, b→ F
27	Which of the following are a tree searching methods? (a)Pre order (b) Post order (c) In order (d) Left order (e) Right Order (f) Central order Identify from below (i) a,b,c, d,e,f (ii) b,c,d (iii) b,c,e (iv) a,b,c (v) None of the above
28	Graphs are shown here with their names below them. Identify the correct one from below (i)a→T, b→ T, c→ T (ii)a→T, b→ F , c→ T (iii) a→T, b→ T, c→ F (iv)a→F, b→ F, c→ F

	(a) 	(b) 	(c) 
	(a) neither an Euler circuit nor an Euler path	(b) no Euler circuit, an Euler path .	(c) No Euler circuit
29	If $m \geq \frac{1}{2}(n^2 - 3n + 6)$ where m be the number of edges and n be number of vertices in a connected graph G then (i) G has Eulerian Circuit (ii) G has Hamiltonian Circuit (iii) G has both Eulerian and Hamiltonian Circuit (iv) None of these		
30	Identify the value of minimum Spanning tree from graph given alongside using Prim algorithm (a) 20 (b) 30 (c) 28 (d) 31		

Section B:

Each Question carry 4 marks

1. Use induction to show that $\prod_{k=1}^{n}\left(1-\frac{1}{2^k}\right) \geq \frac{1}{4}+\frac{1}{2^{n+1}}$," n ≥ 1.

2. Prove the following (i) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ (ii) ((p⇒q)∧(q⇒r))⇒ (p ⇒ r).

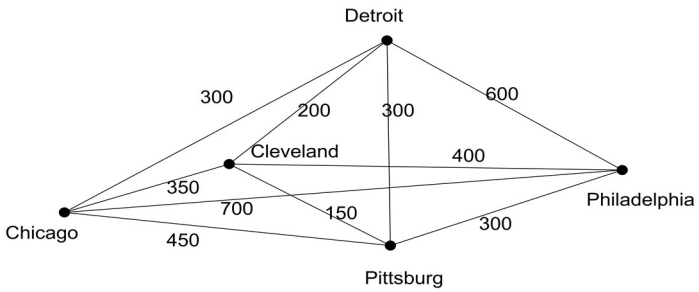
Or If A=(1,2,3,4), B=(a,b,c), R={(1,a), (1,b), (2,b), (2,c), (3,b), (4,a)}. Compute $\overline{R}, R \cap S, R \cup S, \overline{R}$, R^{-1}.

3. Utilize the Generating function in solving the following recurrence relations $a_n - 9a_{n-1} + 26\,a_{n-2} - 24\,a_{n-3} = 0$, given that a_0=1, a_1=1, a_2=10.

Or If A=(1,2,3,4), B=(a,b,c), R={(1,a), (1,b), (2,b), (2,c), (3,b), (4,a)}. Compute $\overline{R}, R \cap S, R \cup S, \overline{R}$, R^{-1}.

4. Show that the (3, 7) encoding function e : $B^3 \rightarrow B^7$ defined by e(000) = 0000000 e(100) = 1000101 e(001) = 0010110 e(101) = 1010011 e(010) = 0101000 e(110) = 1101101 e(011) = 0111110 e(111) = 1111011 is a group code.

5. Find minimal spanning trees using Prim's algorithm or Kruskal's algorithm.



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