BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : II**
BRANCH: MATHEMATICS & COMPUTING **SESSION : SP/2022**

SUBJECT: MA110 COMPLEX ANALYSIS

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) The following function $f(z)$ is given: [5]

$$f(z) = 2x + ixy^2$$

i) Discuss the continuity of the function $f(z)$ in the complex plane.

ii) With the help of Cauchy - Riemann equations, check whether the function $f(z)$ possesses the derivative $f'(z)$ at any point or not.

Q.1(b) Establish that the real - valued function $u(x, y) = 2x(1-y)$ is harmonic. If $f(z) = u(x, y) + iv(x, y)$ is an analytic function, then determine the harmonic conjugate $v(x, y)$. Also, obtain the analytic function $f(z)$ in terms of z. [5]

Q.2(a) Is it possible to apply Cauchy - Goursat theorem to show that $\oint_C f(z)dz = 0$ when the contour C is the circle $|z| = 1$, and when [5]

i) $f(z) = \dfrac{z^2}{z-3}$

ii) $f(z) = ze^{-z}$

are taken as integrands? Support your answer with proper reasoning.

Q.2(b) Using Cauchy's Integral formula or its extension, compute the value of the integral: [5]

$$I = \frac{1}{2\pi i} \oint_C \frac{z^3 + 2z^2 + 5}{(z-1)^4} dz$$

around the circle $C : |z-2| = 3$.

Q.3(a) Obtain the Taylor and Laurent series expansion of the function $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the following regions: [5]

i) $|z| < 1$ ii) $1 < |z| < 2$ iii) $0 < |z-2| < 1$

Q.3(b) For the two functions: [5]

$$f_1(z) = e^z \text{ and } f_2(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 \, (a_3 \neq 0)$$

Establish that, at infinity, $f_1(z)$ has an isolated essential singularity, but $f_2(z)$ has a pole of order three.

Q.4(a) Determine the poles and residues at each pole of the function $f(z) = \dfrac{z-3}{(z^2+4z-5)^2}$. Hence, with the use of Residue theorem, evaluate the integral $\oint_C f(z)dz$, where C is the circle $|z| = 5.5$. [5]

Q.4(b) By choosing an appropriate closed contour C in the upper half plane and applying calculus of residues, evaluate the value of the real integral: [5]

$$\int_0^\infty \frac{\cos ax}{x^2+1} dx, a \geq 0$$

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Q.5(a) Recognize the image in the $w-$plane of the circle $|z-3|=3$ in the $z-$ plane under the inverse mapping $w=\frac{1}{z}$. [5]

Q.5(b) Construct the bilinear transformation $w=f(z)$ that maps the points $z_1=-i, z_2=1$ and $z_3=i$ onto the points $w_1=-1, w_2=0$ and $w_3=1$ respectively. [5]

:::::20/07/2022:::::