

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC
BRANCH: CHEMISTRY/PHYSICS

SEMESTER : II
SESSION : SP/2022

TIME: 3 Hours

SUBJECT: MA108 MATHEMATICS - III

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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Q.1(a) Describe the convergence and divergence of a Geometric series. [5]

Q.1(b) Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ [5]

Q.2(a) Find the solution of the given system of linear equation [5]

$$-5x - 2y + 2z = 14$$

$$3x + y - z = -8$$

$$2x + 2y - z = -3$$

Q.2(b) Show that the matrix A $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ [5]

satisfies the Cayley Hamilton Theorem and hence find the value of the matrix equation
 $-A^3 + 4A^2 + 5A - 2I$

Q.3(a) State and prove Euler's theorem for homogeneous functions of two variables. [5]

Q.3(b) Find the value of $\lim_{(x,y) \rightarrow (0,0)} (1 + x^2 + y^2)^{\frac{1}{x^2+y^2}}$ [5]

Q.4(a) Find the value of $\int_0^1 x^4(1-x)^2 dx$ [5]

Q.4(b) Find the value of the double integral $\int_0^1 \int_x^1 e^{y^2} dy dx$ by reversing the order of integration. [5]

Q.5(a) Apply divergence theorem to evaluate $\iint (x+z)dydz + (y+z)dzdx + (x+y)dxdy$ over the surface of the sphere $x^2 + y^2 + z^2 = 9$. [5]

Q.5(b) Show that the force field $F = ((2xy + z^3), x^2, 3xz^2)$ is conservative and hence find a scalar potential of F . [5]

:::::19/07/2022:::::