BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : II**
BRANCH: MATHS AND COMPUTING **SESSION : SP/22**

SUBJECT: MA105 CALCULUS II

TIME: 03 HOURS **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Find the angle between the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the plane $ax + by + cz + d = 0$. [5]

Q.1(b) Find the distance of the point (1, 2, -4) from the line $\frac{x-3}{2} = \frac{y-1}{-5} = \frac{z+2}{3}$. [5]

Q.2(a) Use a triple integral to determine the volume of the region below $z = 4 - xy$ and above the region in the xy-plane defined by $0 \le x \le 2, 0 \le y \le 1$. [5]

Q.2(b) Evaluate $\int \int_D 4xy - 7\, dA$ using polar coordinates, where D is the portion of $x^2 + y^2 = 2$ in the first quadrant. [5]

Q.3(a) Find the unit vector normal to the surface $x^2 + yz$ at the point (1, 1, 1). [5]

Q.3(b) Find the divergence and curl of the vector $\vec{V} = (xyz)\hat{\imath} + 3x^2y\hat{\jmath} + (xz^2 - y^2z)\hat{k}$. [5]

Q.4(a) Evaluate $\int_C \vec{F} . \overrightarrow{dr}$ where $\overrightarrow{F}\,(x, y) = 3\hat{\imath} + (xy - 2x)\hat{\jmath}$ and C is left half of the circle centered at the origin of radius 4 with counter-clockwise direction. [5]

Q.4(b) Use Divergence Theorem to evaluate $\int \int_S \vec{F}.\overrightarrow{ds}$, where $\vec{F} = yx^2\hat{\imath} + (xy^2 - 3z^4)\hat{\jmath} + (x^3 + y^2)\hat{k}$ and S is the portion of the sphere of radius 4 with $z \le 0$ and $y \le 0$. Note that all three surfaces of this solid are included in S. [5]

Q.5(a) Find the expression for gradient of $\phi(u_1, u_2, u_3)$ in orthogonal curvilinear coordinates. [5]

Q.5(b) Find the expression for divergence of $\vec{F} = F_1\hat{e_1} + F_2\hat{e_2} + F_3\hat{e_3}$ in spherical coordinates. [5]

:::::18/07/2022:::::