BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS:M. Tech. **SEMESTER : II**
BRANCH:EEE **SESSION : SP/22**

SUBJECT:EE555 Statistical Control Theory

TIME: 2 Hrs. **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains questions of total 50 Marks.
2. Candidates may attempt all questions maximum of 50 marks. Some questions also contain options.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

---

1. Define random variable? Mention one example. List the properties of PDF and CDF of a random variable. [4]
2. State and explain the theorem on total probability. [2]
3. The joint PDF of random variables X and Y is given by [5]

$$f_{XY}(x,y) = \begin{cases} 2e^{-(x+y)} & for\, x \geq 0,\ y \geq 0 \\ 0 & otherwise \end{cases}$$

(a) Determine the marginal density function of X and Y.
(b) Determine conditional density function of (i) X given Y (ii) Y given X.

4. If X and Y are two discrete RVs, whose joint density function is given by [5]

$$f(x,y) = \begin{cases} \frac{1}{4}(2x+y) & for\ 0 \leq x \leq 1,\ 0 \leq y \leq 2 \\ 0 & otherwise \end{cases}$$

Find E[X], E[Y], E[XY], E[X^2], E[Y^2], Var(X), Var(Y).

5. Distinguish between random variable and random process. List and explain the important properties of random process. [4]
6. Let z=aX+bY, where X and Y are independent random variables and a and b are arbitrary constants. Find the characteristic function of z. [3]
7. Let $X(t) = Cos(\omega t + \theta), Y(t) = Sin(\omega t + \theta)$, where Θ is a random variable uniformly distributed in [-π, π]. Determine the cross covariance of X(t) and Y(t). [3]
8. Explain stationarity of a random process. Appraise the different types of stationarity. [3]
9. Explain Poisson process. Examine whether the Poisson process is a Martingale or not. [3]
10. Obtain the expression for mean, variance and PSD of the output of a linear time invariant system given a random input. [5]
11. Explain physical realizabilty condition. Obtain the expression of optimum transfer function without regarding physical realizabilty. [5]
12. Explain how the concept of covariance function can be utilized for designing a Kalman filter. [4]
13. Outline and deduce the Markov properties. Give one example of Markov process in any application. Show the state transition diagram also. [4]