

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)**

CLASS: M.Tech.  
BRANCH: EEE

SEMESTER: II  
SESSION: SP/22

SUBJECT: EE553 Nonlinear Control System

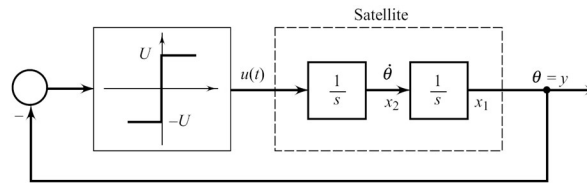
TIME: 2 hrs.

FULL MARKS: 50

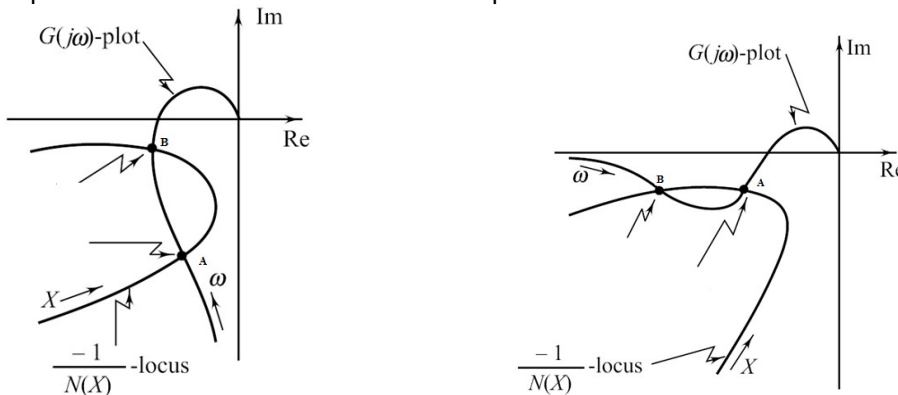
**INSTRUCTIONS:**

1. The question paper contains 6 questions each of 5+5 marks.
2. Attempt any 5 questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Graph paper to be supplied to the candidates in the examination hall.

- Q.1(a) Interpret singular points. Discuss with proper diagram: saddle point, stable and unstable focus, stable and unstable node. [5]  
CO1 L2
- Q.1(b) Apply isocline method to sketch a typical phase trajectory for the following satellite-attitude control system. [5]  
CO2 L4



- Q.2(a) Discuss jump resonance phenomenon in forced nonlinear springs. [5]  
CO2 L3
- Q.2(b) Compute stability analysis applying the concept of describing function to a nonlinear system. Identify and differentiate the stable and unstable limit cycles at A and B for the following plots with proper explanation. Label the limitations of these predictions. [5]  
CO3 L4



- Q.3(a) Apply variable gradient method to examine the stability of the origin of the system described by  $\dot{x}_1 = -x_1 + 2x_1^2x_2$  and  $\dot{x}_2 = -x_2$ . [5]  
CO3 L4
- Q.3(b) Examine the stability of the equilibrium state of the system by direct and indirect methods, described by  $\dot{x}_1 = x_2$  and  $\dot{x}_2 = -x_1 - x_1^2x_2$ . Illustrate that Lyapunov's linearization method fails while the direct method can easily solve this problem. [5]  
CO3 L4
- Q.4(a) Consider the nonlinear control-affine system  $\dot{x} = f(x) + g(x)u$ . Perform transformation of states into linearizable form for the system [5]  
CO3 L4
- a.)  $\dot{x}_1 = a \sin x_2$ ;  $\dot{x}_2 = -x_1^2 + u$ ; and find the control law
  - b.)  $\dot{x}_1 = a \sin x_2$ ;  $\dot{x}_2 = -x_1^2 + u$ ;  $y = x_2$ ; find control law using input-output linearization

4.(b) Define Lie derivative. Find  $L_f h(x), L_g h(x)$  when

[5]  
CO3 L3

$$h(x) = \frac{1}{2}(x_1^2 + x_2^2);$$

$$f(x) = \begin{bmatrix} -x_2 \\ -x_1 - \mu(1 - x_1^2)x_2 \end{bmatrix}; g(x) = \begin{bmatrix} -x_1 - x_1 x_2^2 \\ -x_2 + x_1^2 x_2 \end{bmatrix}$$

Q.5(a) Consider a nonlinear system described by the equations:  $\dot{x}_1 = -3x_1 + x_2; \dot{x}_2 = x_1 - x_2 - x_2^3$   
Using the Krasovskii method for constructing the Lyapunov function with P as identity matrix, investigate the stability of the equilibrium state. Find a region of asymptotic stability.

[5]  
CO4 L4

Q.5(b) Design a Lyapunov function-based control law to stabilize the following nonlinear system.

[5]  
CO5 L5

$$\dot{x}_1 = x_2^3; \dot{x}_2 = u$$

Q.6(a) Define 1-norm, 2-norm, and  $\infty$ -norm for the vector  $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$

[5]  
CO2 L3

Fill in the blanks:

For the existence of a sliding mode on the switching surface, the state velocity vectors should be directed ----- (towards/ away from) the surface, i.e., the system must be ----- (stable/ unstable) to the switching surface.

A system with m inputs can have ----- switching functions and ----- sliding surfaces

Q.6(b) What are the steps in designing a sliding mode controller? Mention the advantages. Why is a sliding mode robust? What is chattering and why is this undesirable?

[5]  
CO2 L2

:29/04/2022: