BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: MTECH | **SEMESTER : II**
BRANCH: EEE | **SESSION : SP/22**

SUBJECT: EE551 OPTIMAL CONTROL THEORY

TIME: 3:00 HOURS | **FULL MARKS: 50**

INSTRUCTIONS:
1. **The question paper contains 5 questions each of 10 marks and total 50 marks.**
2. **Attempt all questions.**
3. **The missing data, if any, may be assumed suitably.**
4. **Before attempting the question paper, be sure that you have got the correct question paper.**
5. **Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.**

Q.1(a) Determine whether the given symmetric matrix is positive definite, positive definite, indefinite or none of these [2]

i) $\begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$ ii) $\begin{bmatrix} -3 & 2 \\ 2 & -6 \end{bmatrix}$

Q.1(b) Determine the nature of the quadratic function $f(x) = 7x_1^2 + 4x_1x_3 + 10x_1x_2 + 5x_2^2 + 8x_2x_3 + 9x_3^2$ using 'Sylvester's Criteria'. [3]

Q.1(c) For functional [5]

$$J(x(t)) = \int_{t0}^{tf} [2x^2(t) + 3x(t) + 4]dt$$

evaluate the first and second variation of the functional.

Q.2(a) [5]

Find the optimum of $J = \int_0^2 [\dot{x}^2(t) - 2tx(t)]dt$

that satisfy the boundary condition X(0)=1 and x(2)=5

Q.2(b) Minimize $f(x) = f(x_1, x_2) = x_1^2 + 4x_2^2 - 4x_1 - 6x_2$ [5]

Subject to constraint $\begin{matrix} x_1 + x_2 = 2 \\ 2x_1 + 2x_2 \le 12 \end{matrix}$; $\begin{matrix} x_1 > 0 \\ x_2 > 0 \end{matrix}$ using KKT condition.

Q.3(a) Find the equation of the curve that is extrimal for the functional [5]

$$J(x) = \int_{t_0}^{t_f} [t\,\dot{x}(t) + \dot{x}^2(t)]dt$$

And $(t_f > 0)$, for the boundary condition specified below:

a) if $t_f = 1, x(0) = 1, x(1) = 2.75$

b) if $x(0) = 1, x(t_f) = 5\, and \quad t_f\ is \quad free$

Q.3(b) Find the extremal for the functional [5]

$$J = \int_{t_0=0}^{t_f} (2\dot{x}^2(t) + 24tx(t))dt$$

Left end point is fixed i.e $x(t_0) = 0$, $t_0 = 0$,is fixed and t_f is free but $x(t_f) = 2$.

Q.4(a) [5]

1. Find the extema using pontrygain principle Consider the control system described by and the Performance Index (J)

$$J = \int_{t0}^{tf} [x_1^2(t) + 2u^2(t)]dt$$

Under the condition

$$\dot{x}_1(t) = x_2(t); \quad t_0 = 0;$$
$$\dot{x}_2(t) = u(t); \quad t_f = 5;$$
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t_0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}(t_f) = \begin{bmatrix} 2 \\ 2 \end{bmatrix};$$

Q.4(b) [5]

For the plant $\dot{x}_1(t) = x_2(t)$, $\dot{x}_2(t) = u(t)$

performance index $J = \frac{1}{2}\int_{t_0}^{t_f} u^2(t)dt$ with initial condition $x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $x(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

find the optimal control and optimal state for the given boundary condition, assume the control and state are unconstrained.

Q.5(a) Derive the Matrix Differential Riccati Equation (MDRE) which happens to be the solution of finite time LQR regulator problem? [5]

Q.5(b) Consider the control system described by [5]

$$\dot{x} - Ax + Bu, \quad where\ A - \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, B - \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and the Performance Index (J) and (Q) given by

$$J = \int_0^{\infty} (x_1^2(t) + x_2^2(t) + u^2(t))\, dt$$

Solve for the optimal control signal u*(t) such that the Performance Index (J) is minimized?

:::::25/04/2022 E:::::