BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : II**
BRANCH: QEDS **SESSION : SP/2022**

SUBJECT: ED111 INTERMEDIATE ANALYSIS

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Show that the sequence of functions $\{f_n(x)\}$, where [5]

$$f_n(x) = nxe^{-nx^2}, x \geq 0$$

is not uniformly convergent on $[0, a], a > 0$.

Q.1(b) Find the value of p for which the series $\sum_{n=1}^{\infty} \frac{2x}{4n^p + x^2 n^q}$ converges uniformly over any finite interval $[a, b]$ for $q \geq 0$. [5]

Q.2(a) Find the radius of convergence and the exact interval of convergence of the power series [5]

$$\sum_{n=1}^{\infty} \frac{n}{(2n+1)^2} (x-2)^{3n}.$$

Q.2(b) Find the Fourier coefficients and Fourier series of the periodic function $f(x)$ in $[-\pi, \pi]$, where $f(x) = x - x^2, -\pi < x < \pi$. Hence deduce that [5]

$$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

Q.3(a) Let $f: [a, b] \to \mathbb{R}$ be bounded and $\alpha: [a, b] \to \mathbb{R}$ be monotonic increasing function. Prove that f is Riemann integrable if and only if for given $\epsilon > 0$, there exists a partition $P \in \mathbb{P}$ such that [5]

$$U(P, f, \alpha) - L(P, f, \alpha) < \epsilon,$$

where $\mathbb{P}$ is the set of all partitions of $[a, b]$.

Q.3(b) Show that the function $[x]$, where $[x]$ is the greatest integer not greater than x is Riemann integrable in $[0,3]$ and find $\int_0^3 [x] dx$. [5]

Q.4(a) If $u = \frac{x^3y^3z^3}{x^3+y^3+z^3} + \ln \frac{xy+yz+zx}{x^2+y^2+z^2}$, then find the value of [5]

$$xu_x + yu_y + zu_z.$$

Q.4(b) Find the maximum and minimum values, if exist, of the function $x^2y^3z^4$ where $2x + 3y + 4z = a, a > 0$. [5]

Q.5(a) Evaluate the integral $\int \int_R e^{-(x+2y)} dx\, dy$, where R is the region in the first quadrant in which $x \leq y$. [5]

Q.5(b) Apply triple integral to find the volume of the solid bounded by $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$. [5]

:::::18/07/2022:::::