# BIRLA INSTITUTE OF TECHNOLOGY
MESRA – 835215, RANCHI, INDIA

UG

Name: ……………………………………………………. Roll No.: ………………………………………..

Branch: ……………………………………………………. Signature of Invigilator: ……………………

Semester: IVth Date: 28/04/2022 (MORNING)

Subject with Code: **EC251 PROBABILITY AND RANDOM PROCESSES**

| Marks Obtained | Section A (30) | Section B (20) | Total Marks (50) |
|---|---|---|---|
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**INSTRUCTION TO CANDIDATE**

1. The booklet (question paper cum answer sheet) consists of two sections. First section consists of MCQs of 30 marks. Candidates may mark the correct answer in the space provided / may also write answers in the answer sheet provided. The Second section of question paper consists of subjective questions of 20 marks. The candidates may write the answers for these questions in the answer sheets provided with the question booklet.
2. The booklet will be distributed to the candidates before 05 minutes of the examination. **Candidates should write their roll no. in each page of the booklet.**
3. Place the Student ID card, Registration Slip and No Dues Clearance (if applicable) on your desk. All the entries on the cover page must be filled at the specified space.
4. Carrying or using of mobile phone / any electronic gadgets (except regular scientific calculator)/chits are strictly prohibited inside the examination hall as it comes under the category of unfair means.
5. No candidate should be allowed to enter the examination hall later than 10 minutes after the commencement of examination. Candidates are not allowed to go out of the examination hall/room during the first 30 minutes and last 10 minutes of the examination.
6. Write on both side of the leaf and use pens with same ink.
7. The medium of examination is English. Answer book written in language other than English is liable to be rejected.
8. All attached sheets such as graph papers, drawing sheets etc. should be properly folded to the size of the answer book and tagged with the answer book by the candidate at least 05 minutes before the end of examination.
9. The door of examination hall will be closed 10 minutes before the end of examination. Do not leave the examination hall until the invigilators instruct you to do so.
10. Always maintain the highest level of integrity. Remember you are a BITian.
11. Candidates need to **submit the question paper cum answer sheets before leaving the examination hall**.

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI**
**(END SEMESTER EXAMINATION)**

**CLASS:BTECH** **SEMESTER: IV**
**BRANCH:ECE** **SESSION: SP/22**

**SUBJECT: EC 251 PROBABILITY AND RANDOM PROCESSES**
**TIME: 2:00 HOURS** **FULL MARKS: 50**

**INSTRUCTIONS:**
1. **The question paper contains 40 questions of total 50 marks.**
2. **Question No.1 to Question No.30 are MCQ type each of 1 mark and total 30 marks.**
3. **Question No.31 to Question No.40 is each of 2 marks and total 20 marks.**
4. **Candidates may attempt all 40 questions maximum of 50 marks.**
5. **For MCQ type questions, select the correct answer out of the options given against each question by putting a Tick Mark (✓) against it.**
6. **The missing data, if any, may be assumed suitably.**

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Q.1 The theorem which states least percentage of values that fall within standard deviations is classified as: [1]
(A) Gaussian theorem
(B) Poisson theorem
(C) Chebyshev's theorem
(D) None of these

Q.2 If $X$ and $Y$ are independent random variables defined by probability density function (pdf) $f_{XY}(x,y)$, the pdf of $Z=(X+Y)$ is [1]
(A) $f_X(x).f_Y(y)$
(B) $\int_{-\infty}^{\infty}\int_{-\infty}^{z-y} f_{XY}(x,y).dx.dy$
(C) $\int_{-\infty}^{\infty} f_{XY}(x,y).dx.dy$
(D) $\frac{f_{XY}(x,y)}{f_X(x).f_Y(y)}$

Q.3 If $\varphi_X(\omega)$ is the complex characteristic function of a random variable $X$, then the $n^{Th}$ moment about origin $E[X^n]$ is given by [1]
(A) $E[X^n]=(j)^n\varphi^n{}_X(0)$
(B) $E[X^n]=(j)^n\varphi^n{}_X(\omega)$
(C) $E[X^n]=\left(\frac{1}{j}\right)^n\varphi^n{}_X(0)$
(D) $E[X^n]=\left(\frac{1}{j}\right)^n\varphi^n{}_X(\omega)$

Q.4 The cross-correlation provides: [1]
(A) Information about the structure of only one signal
(B) Information about the behavior of only one signal in the time domain
(C) The measure of dissimilarities between two signals
(D) The measure of similarities between two signals

Q.5 A random variable $X$ has a continuous uniform distribution over the interval (2, 6). The $P[X\le 4]$ is [1]
(A) 0.3
(B) 0.5
(C) 1.33
(D) None of these

Q.6 If $X$ represents the outcomes, when a fair dice is tossed, choose the correct answer from the given options. Here $E[X]$ is the mean value of $X$ and $M_X(t)$ is the moment generating function (MGF) of $X$. [1]
(A) $E[X]=[M_X{}'(t)]_{t=0}=91/6$
(B) $E[X]=[M_X{}''(t)]_{t=0}=7/2$
(C) $E[X]=[M_X{}'(t)]_{t=0}=7/2$
(D) None of these

Q.7 A sequence of random numbers can be termed convergence if [1]

(A) $(X_n - X_{n-1})$ is less than $\varepsilon$, for all $n$

(B) $|X - X_{n-1}|$ is less than $\varepsilon$, for $n >$ some value

(C) $(X - X_n)$ is less than $\varepsilon$ for $n >$ some value

(D) $(X - X_n)$ is zero for some value of $n$

Q.8 Which of the following distribution function(s) show(s) the memoryless property? [1]

(A) Exponential distribution
(B) Gamma distribution
(C) Poisson distribution
(D) All the above

Q.9 If, P(A)=0.5, P(B)=0.3 and $P(A \cap B) = 0.15$, then $P((A|\bar{B})$ in a two-event space is [1]

(A) 0.15
(B) 0.3
(C) 0.5
(D) 0.85

Q.10 Consider a pdf given by f(x) = k $e^{-ax}$, 0 ≤ x≤ ∞. The P(x ≤ a) can be written as [1]

(A) ≤ 1- 1/$a^2$
(B) ≥ 1- 1/$a^2$
(C) ≤ 1/$a^2$
(D) ≥ 1/$a^2$

Q.11 If $f_{XY}(x, y) = k$ is the probability density function (pdf) of bivariate random variables for $0 \le x \le 1, 0 \le y \le 1$ and 0 otherwise, then $E[XY^2]$ is [1]

(A) 1/6
(B) 1/4
(C) 1/3
(D) 1/2

Q.12 For any random variable $X$, the value of $a$ for which the function $E[(X - a)^2]$ is minimum is [1]

(A) $a = E[X]$
(B) $a = E[X]/2$
(C) $a = E[X^2]$
(D) $a = E[X^2]/2$

Q.13 If $X$ and $Y$ are two random variables, then the variance of $(aX + bY)$, where $a$ and $b$ are constants is [1]

(A) $aVar[X] + bVar[Y] - abCov[X, Y]$

(B) $a^2Var[X] + b^2Var[Y] - 2abCov[X, Y]$

(C) $a^2Var[X] + b^2Var[Y] + 2abCov[X, Y]$

(D) $aVar[X] + bVar[Y] + abCov[X, Y]$

Q.14 There are 5 events whose values are {2, -5, 3, 6, -2} with respective probability value {0.1, 0.2, 0.3, 0.2, 0.2}. The variance is [1]

(A) 15.29
(B) 17.31
(C) 27.12
(D) None of these

Q.15 Consider two random variables '$X$' and '$Y$' with joint-characteristic function $\emptyset_{XY}(\omega_1, \omega_2)$, then find joint p.d.f. $f_{XY}(x, y)$. [1]

(A) $\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \emptyset_{XY}(\omega_1, \omega_2) \cdot e^{-j(\omega_1 x + \omega_2 y)} \cdot d\omega_1 \cdot d\omega_2$

(B) $\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \emptyset_{XY}(\omega_1, \omega_2) \cdot e^{-j(\omega_1 x + \omega_2 y)} \cdot d\omega_1 \cdot d\omega_2$

(C) $2\pi\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \emptyset_{XY}(\omega_1, \omega_2) \cdot e^{-j(\omega_1 x + \omega_2 y)} \cdot d\omega_1 \cdot d\omega_2$

(D) $(2\pi)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \emptyset_{XY}(\omega_1, \omega_2) \cdot e^{-j(\omega_1 x + \omega_2 y)} \cdot d\omega_1 \cdot d\omega_2$

Q.16 A random process is called 'white' if the power spectral density is equals to [1]

(A) Symmetric and constant with frequency
(B) Asymmetric and impulse with frequency
(C) Symmetric and impulse with frequency
(D) Asymmetric and constant with frequency

Q.17 The joint CDF of a two-dimensional random variable is $F_{XY}(x, y)$. The $P(X > a, Y > c)$ can be given by [1]

(A) $1 + F_X(a) + F_Y(c) + F_{XY}(a, c)$
(B) $1 + F_X(a) - F_Y(c) - F_{XY}(a, c)$
(C) $1 - F_X(a) - F_Y(c) + F_{XY}(a, c)$
(D) $1 - F_X(a) - F_Y(c) - F_{XY}(a, c)$

Q.18 If A and B are two events making complete space, then $P\left(\bar{A}/_{\bar{B}}\right)$ is [1]

(A) $\frac{1-P(A \cup B)}{P(\bar{B})}$
(B) $1 - P(\bar{A}/_{\bar{B}})$
(C) $1 - P(A/_B)$
(D) None of the above

Q.19 Consider a probability space whose pdf is given by f(x) = k $e^{-3x}$ for x≥0, is equal to 0 otherwise. Find the mean of the random variables. [1]

(A) 3
(B) 1/3
(C) 1/6
(D) 1/9

Q.20 For two events $A$ and $B$ of a two-event space, $P(A \cap B) + P(A \cap \bar{B})$ is represented by [1]

(A) $P(A)$
(B) $P(B)$
(C) $P(A \cup B)$
(D) $P(A \cap B)$

Q.21 A random variable $X$ has mean of 9 and variance of 3. Use Chebyshev's inequality and find an upper bound for $P(|X - 9| \geq 3)$. [1]

(A) 1/2
(B) 1/3
(C) 2/3
(D) 3/5

Q.22 Consider a Gaussian pdf with mean and standard deviation equal to 2 and $\sqrt{2}$ respectively. The P(|x-2| ≥ 0.001) can be written as [1]

(A) ≤ 2000
(B) ≥ 2000
(C) ≤ 1414
(D) ≥ 1414

Q.23 When joint probability density function (pdf) of $X$ and $Y$ is $f_{XY}(x, y)$, then cumulative distribution function (CDF) of $z = (X^2 + Y^2)$ can be given as $F_Z(z)$ equals to [1]

(A) $\int_{-\sqrt{z}}^{\sqrt{z}} \int_{-(z-y)}^{(z-y)} f_{XY}(x, y).dx.dy$
(B) $\int_{0}^{\sqrt{z}} \int_{-\sqrt{(z-y)}}^{\sqrt{(z-y)}} f_{XY}(x, y).dx.dy$
(C) $\int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{(z-y^2)}}^{\sqrt{(z-y^2)}} f_{XY}(x, y).dx.dy$
(D) $\int_{0}^{z} \int_{-\sqrt{(z-y^2)}}^{\sqrt{(z-y^2)}} f_{XY}(x, y).dx.dy$

Q.24 The auto-correlation function of a band-limited ($B\ Hz$) white-noise process is given by: [1]

(A) $R(\tau) = B.N_0.Ln(2B\tau)$
(B) $R(\tau) = B.N_0.\tau.\sin(2B)$

(C) $R(\tau) = B.N_0.Log(2B\tau)$

(D) $R(\tau) = B.N_0.\text{sinc}(2B\tau)$

Q.25 If 'A' and 'B' represents space of two events, then the probability that exactly one of them occurs is [1]

(A) $P(A) + P(B)$
(B) $P(A) + P(B) - P(A \cap B)$
(C) $P(A) + P(B) - 2P(A \cap B)$
(D) $P(\bar{A}) + P(\bar{B})$

Q.26 The auto-correlation function of a wide-sense stationary random process is given as $e^{-2|\tau|}$. Find the peak value of the spectral density. [1]

(A) 1
(B) 2
(C) $e$
(D) $e^{-1/2}$

Q.27 Let $X$ is a random variable and $B$ is the conditioning event defined as $B = (X \leq b)$, where $b$ is some real number $-\infty < b < \infty$, also $F_X(x)$ and $f_X(x)$ denote cumulative distribution function (CDF) and probability distribution function (pdf) of $X$ respectively, then conditional density function $f_X(x|X \leq b)$ for $X < b$ is [1]

(A) $\frac{F_X(x)}{\int_{-\infty}^{b} f_X(x).dx}$

(B) $\frac{F_X(x)}{\int_{0}^{b} f_X(x).dx}$

(C) $\frac{f_X(x)}{\int_{-\infty}^{b} f_X(x).dx}$

(D) $\frac{f_X(x)}{\int_{0}^{b} f_X(x).dx}$

Q.28 Tickets numbered 1 to 20 are mixed-up and then a ticket is drawn at random. Find the probability that the ticket drawn has a number which is a multiple of 5. [1]

(A) 1/4
(B) 1/5
(C) 8/15
(D) 9/20

Q.29 If $X$ be the sum of *N* uncorrelated random variables, the covariance $C_{X_iX_j}$ of $X$ can be given for large *N* by [1]

(A) $\sigma_{X_i}{}^2; for\ i = j$

(B) $N\sigma_{X_i}{}^2; for\ i = j$

(C) $\frac{N\sigma_{X_i}{}^2}{2}; for\ i = j$

(D) $\frac{\sigma_{X_i}{}^2}{N}; for\ i = j$

Q.30 If the co-variance of two random variables $X$ and $Y$ is $C_{XY}$, find their correlation coefficient $\rho_{XY}$. [1]

(A) $Ln[C_{XY}]$

(B) $Ln\left(\frac{C_{XY}}{\rho_X.\rho_Y}\right)$

(C) $\frac{\rho_X.\rho_Y}{C_{XY}}$

(D) $\frac{C_{XY}}{\rho_X.\rho_Y}$

Q.31 State and explain Bayes' Rule for probability. [2]

Q.32 In a class, 60% of the students are boys and the remaining are girls. It is known that the probability of a boy getting distinction is 0.30 and that of girl getting distinction is 0.35. Find the probability that a student chosen at random will get distinction? [2]

Q.33 Define moment generating function (MGF) of ensemble of random variables with an example. [2]

Q.34 For a Binomial distribution mean is 6 and standard deviation is $\sqrt{2}$. Find the probability for one success. [2]

Q.35 For a bivariate probability density function (pdf), write expressions for the third moment. [2]

Q.36 Statistically independent random variables $X$ and $Y$ have joint moments as, $m_{10} = 2$, $m_{20} = 16$, $m_{02} = 30$ and $m_{11} = -10$. Find variance of $X$ and $Y$. [2]

Q.37 State and prove Markov's inequality theorem. [2]

Q.38 Verify, if there exist a variate $X$ for which $P[\mu_X - 2\sigma \leq X \leq \mu_X + 2\sigma] = 0.6$. Use Chebyshev's inequality. [2]

Q.39 Explain wide sense stationary (WSS) random process. [2]

Q.40 A wide sense stationary process $X(t)$ has power spectrum $S_{XX}(\omega) = \frac{9}{\omega^2+4}$. Find the average power of $X(t)$. [2]

**:::::28/04/2022:::::**