## SUBJECT: Finite Element Method (CE506)

TIME: 2 Hours
FULL MARKS: 50

## INSTRUCTIONS:

1. The question paper contains 10 questions each of 5 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
Q. 1 The plane truss shown in Figure 1 is composed of members having a cross sectional area of $0.2 \mathrm{~m}^{2}$ and modulus of elasticity $\mathrm{E}=75 \mathrm{GPa}$. Compute the element stiffness matrix of member 1 in the global coordinate system.


Figure 1
Q. 2 Displacement field for three-nodded bar element (shown in Figure 2) is given by [5] $u=N_{1} u_{1}+N_{2} u_{2}+N_{3} u_{3}$. Derive the shape functions $\left(N_{1}, N_{2}, N_{3}\right)$ for the three-nodded bar element.


Figure 2
Q. 3 Show that the displacement field given in the Q. 2 includes the required capability to represent the rigid body deformation of bar. Use the shape functions derived in Q2.
Q. 4 Calculate the equivalent nodal load vector for the beam given in Figure 3. Interpolation functions for two-nodded beam element is given by
$N_{1}=1-\frac{3 x^{2}}{L^{2}}+\frac{2 x^{3}}{L^{3}}, \quad N_{2}=x-\frac{2 x^{2}}{L}+\frac{x^{3}}{L^{2}}, \quad N_{3}=\frac{3 x^{2}}{L^{2}}-\frac{2 x^{3}}{L^{3}}, \quad N_{4}=-\frac{x^{2}}{L}+\frac{x^{3}}{L^{2}}$


Figure 3
Q. 5 Use Galerkin's method of weighted residuals to obtain an approximate solution of the differential equation

$$
\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}+y=2 x \quad 0 \leq x \leq 1
$$

With boundary condition $y(0)=0, y(1)=0$.
Q. 6 Derive the polynomial form of the shape functions numbered $N_{1}$ and $N_{5}$ for the following element

Q. 7 Derive the Jacobian matrix for the isoperimetric mapping of linear element (shown in the figure)

Q. 8 Evaluate the following integral using 2-point Gauss quadrature:

$$
\int_{-1}^{1} \int_{-1}^{1}\left(1+2 x+3 x^{2} y\right) \mathrm{d} x \mathrm{~d} y
$$

Q. 9 Explain the following steps in context of any commercial FE Application:
a) Pre-Processing
b) Analysis
c) Post-processing
Q. 10 Write short notes on plane stress and plane strain idealization.

