BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE **SEMESTER: IV**
BRANCH: CHEMICAL **SESSION : SP/2020**

SUBJECT: CL210 TRANSPORT PHENOMENA

TIME: 2 HOURS **FULL MARKS: 25**

INSTRUCTIONS:
1. The total marks of the questions are 25.
2. Candidates may attempt for all 25 marks.
3. Before attempting the question paper, be sure that you have got the correct question paper.
4. The missing data, if any, may be assumed suitably.

Q1 A fluid of constant density is flowing in laminar flow at steady state in the horizontal x direction between two flat and parallel plates. The distance between the two plates in the vertical y direction is $2y_0$. Derive the equation for the velocity profile within this fluid and the maximum velocity for a distance L m in the x direction. [5]

Q2 For a layer of liquid flowing in laminar flow in the z direction down a vertical plate or surface, derive the velocity profile. Where δ is the thickness of the layer, x is the distance from the free surface of the liquid toward the plate and v_z is the velocity at a distance x from the free surface. [5]
(i). What is the maximum velocity $v_{z,max}$?
(ii). Derive the expression for the average velocity $v_{z\,av}$ and also relate it to $v_{z,max.}$

Q3 A Newtonian fluid of constant density and viscosity is in a vertical cylindrical container of radius R. The container is caused to rotate about its own axis at an angular velocity ω. Find the shape of the free surface of the liquid when steady state has been established. [5]

Q4 An incompressible, isothermal Newtonian fluid is held between two vertically placed co-axial cylinders. Determine the velocity distributions for the flow of the fluid when-the outer cylinder is rotating at an angular velocity ϖ while the inner cylinder is stationary. Use Navier-Stoke's equation. [5]

Q5 Briefly describe the following terms [5]
(i) Gradient; (ii) Divergence; (iii) Substantial time derivative;

§B.4 THE EQUATION OF CONTINUITY[a]

$$[\partial\rho/\partial t + (\nabla \cdot \rho\mathbf{v}) = 0]$$

Cartesian coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-1)}$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-2)}$$

Spherical coordinates (r, θ, φ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \qquad \text{(B.4-3)}$$

[a] When the fluid is assumed to have constant mass density ρ, the equation simplifies to $(\nabla \cdot \mathbf{v}) = 0$.

§B.6 EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT ρ AND μ

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r \qquad \text{(B.6-4)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta \qquad \text{(B.6-5)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad \text{(B.6-6)}$$