BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: M.Tech **SEMESTER: II**
BRANCH: EEE **SESSION: SP/19**

SUBJECT: EE555 STATISTICAL CONTROL THEORY

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Differentiate between PDF and CDF of a random variable. [5]
A fair coin is tossed n times. Let the random variable Y be the difference between the number of heads and the number of tails. Sketch the CDF of Y for n=4.

Q.1(b) Determine and plot $F_Y(y)$ and $f_Y(y)$, if y=-4x+3 and $f_X(x) = 2e^{-2x}U(x)$. [5]

Q.2(a) The joint PDF of random variables X and Y is given by [5]

$$f_{XY}(x,y) = \begin{cases} 2e^{-(x+y)} & for\, x \geq 0,\ y \geq 0 \\ 0 & otherwise \end{cases}$$

(a) Determine the marginal density function of X and Y.
(b) Determine conditional density function of (i) X given Y (ii) Y given X.

Q.2(b) Correlate the PDF with characteristic function. [5]
Let z=aX+bY, where X and Y are independent random variables and a and b are arbitrary constants. Determine the characteristic function of z.

Q.3(a) Compare wiener process and random walk process. [5]
Derive the autocorrelation function of random walk process. Investigate that the random walk process is martingale or not.

Q.3(b) Formulate the steps of designing a real time Wiener filter. [5]
A system has

$$\phi_{ss}(s) = \frac{36}{(s^2-1)(s^2-36)},\ \phi_{nn}(s) = 0.5,\ G_d(s) = e^{0.1s}$$

Determine the optimum overall transfer function using Bode and Shannon method.

Q.4(a) What is power spectral density? Explain. Relate power spectral density to autocorrelation function. [5]
The input into a filter is zero-mean white noise with noise power density N0/2. The filter has transfer function

$$H(f) = \frac{1}{1 + j2\pi f}$$

(a) Compute $S_{YY}(f)$ and $R_{YY}(\tau)$.
(b) Determine the average power of the output.

Q.4(b) Outline how the response of LTIVC system to random inputs is obtained. Derive the expression for mean, variance and PSD of the output in terms of input. [5]

Q.5(a) Outline the properties of transition probability matrix in Markov model? Sketch a state transition diagram for the Markov chains with the following transition probability matrix [5]

$$(a)\begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix} \qquad (b)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Q.5(b) A certain part of machine can be in two parts: working or undergoing repair. A working part fails during the course of a day with probability a. A part undergoing repair is put into working order during the course of a day with probability b. Let X_n be the state of the part [5]
(a) Show that X_n is a two-state Markov chain and give its one-step transition probability matrix P.
(b) Determine the n-step transition probability matrix P^n.