BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: ME **SEMESTER : II**
BRANCH: EEE **SESSION : SP/19**

SUBJECT: EE551 OPTIMAL CONTROL THEORY

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Define Optimization? Classify different types of optimization problems and mention different techniques to solve optimization problems? [5]

Q.1(b) Calculate gradient vector for the function [5]

$$f(x) = x_1^2 + 2x_1x_2 + 4x_1x_3 + 3x_2^2 + 3x_2x_3 + 2x_3^2$$

$$at \quad x = x^* = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Q.2(a) [5]

Find the optimum of $J = \int_0^2 [x^2(t) - 2tx(t)]dt$

that satisfy the boundary condition X(0)=1 and x(2)=5

Q.2(b) [5]

Find the optimum of $J = \int_0^2 [x^2(t) - 2tx(t)]dt$

that satisfy the boundary condition X(0)=1 and x(2)=5

Q.3(a) Find the equation of the curve that is extremal for the functional [5]

$$J(x) = \int_{t_0}^{t_f} [t\,x(t) + x^2(t)]dt$$

And $(t_f > 0)$, for the boundary condition specified below:

a) if $t_f = 1, x(0) = 1, x(1) = 2.75$

b) if $x(0) = 1, x(t_f) = 5 \; and \quad t_f \; is \quad free$

Q.3(b) Find the extremal for the functional [5]

$$J = \int_{t_0=0}^{t_f} (2x^2(t) + 24tx(t))dt$$

Left end point is fixed i.e $x(t_0) = 0$, $t_0 = 0$, is fixed and t_f is free but $x(t_f) = 2$.

Q.4(a) Outline the algorithmic steps to compute optimal control u*(t) using Hamilton–Jacobi-Bellman (HJB) approach? [5]

Q.4(b) Determine the smooth curve of smallest length connecting point $x(0) = 1$ to the line $t = 5$. [5]

The length of the curve lying in the $t - x(t)$ plane, with $t_0 = 0$ and $t_f = 5$

$$J(x) = \int_0^5 [1 + x^2(t)]^{1/2} dt$$

PTO

Q.5(a) State the objective of Infinite final time LQR problem and discuss about the stability analysis of the closed loop system for Infinite time LQR problem? [5]

Q.5(b) Consider the control system described by [5]

$$\dot{x} = Ax + Bu, \qquad where\ A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and the Performance Index (J) and (Q) given by

$J = \int_0^\infty (x_1^2(t) + x_2^2(t) + u^2(t))\, dt$

Solve for the optimal control signal u*(t) such that the Performance Index (J) is minimized?

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