BRANCH: CHEM. ENGG. / CEP\&P

SEMESTER : IV
SESSION : SP/19

## SUBJECT: CL4005 NUMERICAL METHODS FOR CHEMICAL ENGINEERS

TIME: $\quad 3$ Hours
FULL MARKS: 60

## INSTRUCTIONS:

1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
Q.1(a) Graphically explain the Bisection method, Regula Falsi method and Newton-Raphson method in details.
Q.1(b) For turbulent flow of a fluid in a hydraulically smooth pipe, Prandtl's universal resistance law relates the friction factor $f$ and the Reynold's number Re, according to the following relationship

$$
\frac{1}{\sqrt{f}}=-0.40+4 \log _{10}(R e \sqrt{f})
$$

Compute $f$ for $\operatorname{Re}=10^{\wedge 5}$, using the Bisection method lower guess 0.002 , upper guess 0.01 . Do three iterations Report your results in tabular form.
Q.1(c) A circle of radius 2 units is intersected at two points, $A$ and $B$ by the curve, $e^{x}+y=1$. Using an appropriate numerical technique, determine the co-ordinates of point $A$ OR point $B$. Report all intermediate values in a tabular form as explained in the class.
Q.2(a) Compute inverse matrix of the following matrix using LU decomposition method.

$$
\left[\begin{array}{ccc}
1 & -1 & 2 \\
2 & -2 & 3 \\
1 & 1 & 1
\end{array}\right]
$$

Q.2(b) The following equations is obtained after a finite difference technique is applied to a particular problem

$$
\begin{gathered}
x_{1}+x_{2}+x_{3}=6 \\
2 x_{1}+3 x_{2}+4 x_{3}=20 \\
3 x_{1}+4 x_{2}+2 x_{3}=17
\end{gathered}
$$

Solve these linear set of equations using Gauss-Jordan technique and also obtain the inverse of the coefficient matrix.
Q.3(a) $\left(x_{n}, y_{n}\right),\left(x_{n-1}, y_{n-1}\right)$ and $\left(x_{n-2}, y_{n-2}\right)$ using these data points (equispaced base points) along Newton's backward differences prove that

$$
y(x)=y_{n}+\alpha \nabla y_{n}+\frac{\alpha(\alpha+1) \nabla^{2} y_{n}}{2!}, \text { where } \alpha \text { is defined as } \alpha=\frac{x-x_{n}}{\Delta x}
$$

Q.3(b) Find the Newton divided difference polynomial which interpolates the points $(1.6,2),(2,8),(4,8)$, (4.5, 2).
Q.3(c) Write a pseudo programming code for the Newton divided difference polynomial.
Q.4(a) Derive the formula for $\mathrm{a}_{0}$ and $\mathrm{a}_{1}$ for linear fitting of set of data.
Q.4(b) It is known that the tensile strength of a plastic increases as a function of the time it is heat treated. The following data are collected

| Time | 10 | 15 | 20 | 25 | 40 | 50 | 55 | 60 | 75 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Tensile strength | 5 | 20 | 18 | 40 | 33 | 54 | 70 | 60 | 78 |

Fit a straight line to this data and use the equation to determine the tensile strength at a time of 32 min.
Q.4(c) The following table gives the effect of the aromatics concentration, $C_{A}$, on the rate, $r_{A}$, of coke formation on a metal plate during pyrolysis of naphatha in a jet stirred reactor, at 1083K. Determine the order of the reaction using the expression $r_{A}=k C_{A}^{n}$. Also determine the regression coefficient.

| $10^{4}{ }^{*} \mathrm{C}_{\mathrm{A}}\left(\mathrm{kmol} / \mathrm{m}^{3}\right)$ | 1.79 | 2.03 | 2.22 | 2.47 | 2.97 | 3.39 | 4.95 | 7.37 | 9.01 | 9.83 | 10.07 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $10^{2}{ }^{*} \mathrm{r}_{\mathrm{A}}\left(\mathrm{kg} / \mathrm{m}^{2}-\mathrm{hr}\right)$ | 0.28 | 0.32 | 0.36 | 0.40 | 0.49 | 0.59 | 0.99 | 1.55 | 2.00 | 2.25 | 2.60 |

Q.5(a) Prove that the trapezoidal rule is given by $\int_{x_{0}}^{x_{n}} y d x=\frac{h}{2}\left(y_{0}+y_{1}+y_{3}+\cdots+y_{n}\right)$
Q.5(b) Velocity vs. time data for a car are given below. Find distance traversed by the car using a suitable numerical scheme.

| Velocity (km/hr) | 0 | 22 | 30 | 27 | 18 | 7 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Time (hr) | 0 | 2 | 4 | 6 | 8 | 10 | 12 |

Q.5(c) Radial velocity profile of a water flow in a pipe ( $\mathrm{R}=5 \mathrm{~cm}$ ) is given in the table below:

| Radial distance from center, $\mathrm{r}(\mathrm{cm})$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Velocity, $\mathrm{u}(\mathrm{cm} / \mathrm{s})$ | 4 | 3.84 | 3.36 | 2.56 | 1.44 | 0 |

Q.6(a) Given $\frac{d y}{d x}+\frac{y}{1+x}=0$ subjected to $y(0)=2$. Find the value of $y(1.5)$ using Runge-Kutta $4^{\text {th }}$ order method. Take $h=0.5$.
Q.6(b) Solve the boundary value problem $\frac{d^{2} y}{d x^{2}}+\frac{4 x}{1+x^{2}} \frac{d y}{d x}+\frac{2}{1+x^{2}} y=0$, with boundary condition $y(0)=1$ and $y(2)=0.2$. Take $h=0.5$.
Q.6(c)
Q.7(a) Exemplify parabolic and elliptic equations.
Q.7(b) Briefly describe Neumann boundary condition for the solution of boundary value problem.
Q.7(c) Solve the Laplace equation, $\frac{\partial^{2} T}{\partial x^{2}}+\frac{\partial^{2} T}{\partial y^{2}}=0$, for all 4 interior nodes of the figure below. The boundary conditions in each face are shown in the figure. Take $\Delta x=\Delta y=1$;

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