BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS:IMSC **SEMESTER : IX**
BRANCH: Mathematics and Computing **SESSION : MO/2025**

SUBJECT: MA501 FUNCTIONAL ANALYSIS

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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Q.1(a) State and Prove Holder's inequality. [5] 1 2

Q.1(b) Let X be a metric space. Let E and F be disjoint nonempty closed subsets of X. Prove that there is a continuous function f$: X \to [0,1]$ such that $f|_E$=0 and f_F=1. [5] 1 3

Q.2(a) Let *X* be a normed space and *Y* be a subspace of *X* with $Y \neq X$. Let *r* be a real number such that $0 < r < 1$. Prove that there exists some *r >0* such that $||x_r|| = 1$ and $r < dist(x_r, Y) \leq 1$. [5] 2 3

Q.2(b) Let *X* and Y be a normed spaces and $F: X \to Y$ be a linear map. If *F* is continuous on *X*, then prove that *F* is uniformly continuous on X. [5] 2 3

Q.3(a) State and prove closed graph theorem. [5] 3 2

Q.3(b) Let *X* be a *nls* over *K* and *E₁* and *E₂* nonempty disjoint conves subsets of X with *E₁* open. Prove that there exist $f \in X'$ and α $\in R$ such that $Ref(x_1) < \alpha \leq Ref(x_2)$ for all $x_1 \in E_1$ and $x_2 \in E_2$. [5] 3 3

Q.4(a) Let <.,> be an inner product on a linear space *X*. For $x \in X$, let $||x||$ denote a non-negative square root of $< x, x >$. Then prove that $| < x, y > \leq ||x|| \, ||y||$ for every $x, y \in X$, where equality holds if and only if *x* and *y* are linearly dependent. [5] 4 3

Q.4(b) State and prove Polarization identity in an inner product space. [5] 4 3

Q.5(a) Let $\{x_1, x_2, \ldots, x_n\}$ be an orthogonal set in an inner product space *X*. The prove that $||x_1 + x_2 + \cdots + x_n||^2 = ||x_1||^2 + ||x_2||^2 + \cdots + ||x_n||^2$. [5] 2

Q.5(b) Let $\{u_1, u_2, \ldots, u_m\}$ be an orthonormal set in X. Then prove that for every $x \in X$, [5] 3

$$\sum_{n=1}^{m} < x, u_n > |^2 \leq ||x||^2,$$

Where the equality holds iff $x \in span\{u_1, u_2, \ldots, u_m\}$.

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