BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC | **SEMESTER : VII**
BRANCH: MATHEMATICS & COMPUTING | **SESSION : MO/2025**

SUBJECT: MA402 ADVANCED COMPLEX ANALYSIS

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.	Question	Marks	CO	BL
Q.1(a)	State and prove Liouville's theorem. Hence, use it to determine whether the function $f(z) = \cos z$ is a bounded function or not.	[5]	1	2.3
Q.1(b)	Using Cauchy - Goursat theorem for multiple connected domain, evaluate the integral $\oint_C \frac{1}{(z^2-9)} dz$, if $C: \lvert z\rvert = 4$ be positively oriented circle.	[5]	1	3
Q.2(a)	Discuss conformal mapping for the transformation $w = f(z)$ from the $z-$ plane to the $w-$ plane. Find the critical points where the following transformation fails to be conformal: $w = f(z) = z^3 - 3z^2 + 2$	[5]	1	2
Q.2(b)	Develop the Taylor/Laurent series expansions of the function $f(z) = \frac{1}{(z-1)(z-2)}$ in the following regions: i) $\lvert z\rvert < 1$ ii) $1 < \lvert z\rvert < 2$ iii) $\lvert z\rvert > 2$	[5]	2	2,3
Q.3(a)	Let γ be a simple closed contour with positive orientation and enclosing the origin, then, using residues, show that: $\oint_\gamma e^{\left(z+\frac{1}{z}\right)} dz = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!\,(n+1)!}$	[5]	3	2
Q.3(b)	Check whether the function $f(z) = (z^2 - 16)^{\frac{1}{2}}$ is a multivalued function or not. If it is so, obtain the branch point(s) and a suitable branch cut for it.	[5]	3	1,2
Q.4(a)	If a function $\boldsymbol{f(z)}$ is meromorphic inside a simple closed contour $\boldsymbol{C}$ and $\boldsymbol{f(z)}$ is analytic and has no zeros on $\boldsymbol{C}$, then prove that: $\frac{1}{2\pi} \oint_C \frac{f'(z)}{f(z)} dz = N - P$ (1) where N is the number of zeros and P is the number of poles lying inside C (a pole or zero of order m must be counted m times). Using the above expression (1), compute the value of integral $\oint_C \frac{f'(z)}{f(z)} dz$ with $f(z) = \frac{(z-1)^3}{z^4(z-4)^2}$, where $\boldsymbol{C}$ is the circle $\lvert z\rvert = 3$. Also, determine the winding number of the transformation	[5]	3	2,3
Q.4(b)	State Rouche's theorem. Applying Rouche's theorem, show that none of the zeros of the polynomial $P(z) = z^8 - 4z^3 + 12$ lie inside the circle $\lvert z\rvert = 1$.	[5]	2	1,3

PTO

Q.5(a)	For an entire function $f(z)$ given in the Weierstrass Factorization form as: $$f(z) = z^4\ e^z \prod_{n=1}^{\infty}\left(1-\frac{z}{n}\right)\ e^{\frac{z}{n}+\frac{1z^2}{2n^2}}$$ i) Identify all the zeros of $f(z)$ along with their multiplicities. ii) Recognize the canonical product and its genus. iii) Obtain the rank and genus of function $f(z)$.	[5]	3	2,3
Q.5(b)	Explain order of an entire function. Hence, identify the order of the function $f(z) = e^z$.	[5]	3	1,2

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