BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc **SEMESTER : VII**
BRANCH: Mathematics & Computing **SESSION : MO/2025**

SUBJECT: MA401, REAL ANALYSIS AND MEASURE THEORY

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	**BL**
Q.1(a)	Prove that a function f of bounded variation on $[a,b]$ can be expressed as the difference of two monotone increasing functions.	[5]	1	3
Q.1(b)	Suppose f, g are of bounded variation on $[a,b]$. Show that $f+g$ is also of bounded variation on $[a,b]$.	[5]	1	1
Q.2(a)	Consider the real valued function $f: [0,5] \to \mathbb{R}$ defined by $f(x) = [x]$, greatest integer function less than or equal to x. Prove that f is Riemann integrable and find the value of the integral.	[5]	2	3
Q.2(b)	Let $f(x) = x^3$ on $[0,2]$. Is f Riemann integrable? If so, find the value of the integral $\int_0^2 f(x)dx$.	[5]	2	1
Q.3(a)	Show that a set whose outer measure is zero is measurable. Hence, conclude that every countable set is measurable.	[5]	3	1
Q.3(b)	Suppose f, g are measurable functions defined over a measurable set E. Prove that $\{x \in E: f(x) > g(x)\}$ is measurable.	[5]	3	3
Q.4(a)	Let f, g be bounded measurable functions on a set of finite measure E and $f = g$ almost everywhere. Prove that $\int_E f = \int_E g$.	[5]	4	3
Q.4(b)	Suppose f is a non-negative measurable function on E and $\int_E f = 0$. Show that $f = 0$ almost everywhere on E.	[5]	4	1
Q.5(a)	State Fatou's Lemma and use it to prove the monotone convergence theorem.	[5]	5	3
Q.5(b)	Suppose $\{f_n\}$ is a sequence of non-negative measurable functions on a measurable set E that converges pointwise on E to the function f. Suppose $f_n \le f$ on E for each $n \in \mathbb{N}$. Prove that $\lim_{n\to\infty} \int f_n = \int f$ over E.	[5]	5	3

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