BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : V/ADD**
BRANCH: MATHEMATICS **SESSION : MO/2025**

SUBJECT: MA301R1 / MA301 PROBABILITY AND STATISTICS

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

Q.1(a) Let X be a random variable having probability density function [2]

$$f(x) = \begin{cases} c|x(2-x)|, & -2 < x < 1 \\ 0, & otherwise \end{cases}$$

where c is a real constant. Find the value of c.

Q.1(b) The density function of a random variable X is given by [3]

$$f(x) = \begin{cases} a + bx^2, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

If $E[X] = \frac{3}{5}$, find a and b.

Q.2(a) Consider A and B are two events. If $P(A) = 0.9$ and $P(B) = 0.9$, show that $P(A \cap B) \ge 0.8$. [2]

Q.2(b) A pair of fair dice is rolled. Let A denote the event that the sum of the numbers on the dice is equal to 7. [3]

(i) Show that A is independent of the event that the first die lands on 4

(ii) Show that A is independent of the event that the second die lands on 3.

Q.3(a) Let X and Y are two random variables with $E[X^2] < \infty$ and $E[Y^2] < \infty$. Show that $(E[XY])^2 \le E[X^2]E[Y^2]$. [2]

Q.3(b) A student is given a multiple-choice exam with 10 questions, each question with five possible answers. Assume the student randomly chooses answers to the questions. [3]

(i) Find the probability that the student will get exactly 3 questions correct.

(ii) Find the probability that the student will get atleast 3 questions correct.

Q.4(a) A company has two plants to manufacture motorcycles. 70% of the motorcycles are manufactured at the first plant, while 30% are manufactured at the second plant. At the first plant, 80% of the motorcycles are rated as standard quality, while at the second plant, 90% are rated as standard quality. A motorcycle is randomly selected and found to be of standard quality. Find the probability that it was manufactured at the second plant. [2]

Q.4(b) A communications system consists of n components, each of which will, independently, function with probability p. The total system will be able to operate effectively if at least one-half of its components function. For what values of p is a 5-component system more likely to operate effectively than a 3-component system? [3]

Q.5(a) Let X be a Poisson random variable with mean $\lambda > 0$. Compute the mean (or expectation) of the random variable $X!$. (Note that $n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$). [2]

Q.5(b) Let X and Y be two jointly continuous random variables with the following joint probability density function (PDF) [3]

$$f(x,y) = \begin{cases} x + cy, & 0 \le x \le 1, x + y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the constant c that makes $f(x,y)$ a valid joint PDF. Also, find $P\left(0 \le X \le \frac{1}{2}, 0 \le Y \le \frac{1}{2}\right)$.

:::::19/09/2025 :::::E