

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC
BRANCH: MATHEMATICS

SEMESTER : V
SESSION : MO/2025

SUBJECT: MA301R1 PROBABILITY AND STATISTICS

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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Q.1(a) Let X be a continuous random variable having the probability density function (PDF) [5] CO BL

$$f(x) = \begin{cases} cx^2e^{-x}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$

- (i) Find the value of c .
 - (ii) Derive the cumulative distribution function $F(x)$.
 - (iii) Find $P(0 < X \leq 1)$.
- Q.1(b) Suppose that n independent trials, each of which results in any of the outcomes 0, 1 and 2, with respective probabilities 0.3, 0.5 and 0.2, are performed. Find the probability that both outcome 1 and outcome 2 occur at least once. [5]

Q.2(a) Let X be a discrete random variable with the following PMF [5]

$$P_X(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Find the range R_X , the range of the random variable X
 - (ii) Find $P(X \leq 0.5)$.
 - (iii) Find $P(0.25 \leq X \leq 0.75)$.
 - (iv) Find $P(X = 0.2 | X < 0.6)$.
- Q.2(b) Let X represent the number that occurs when a green die is tossed and Y the number that occurs when a red die is tossed. Find the variance of the random variable (a) $2X - Y$, (b) $X + 3Y - 5$. [5]

Q.3(a) Let X and Y be two jointly continuous random variables with the following joint probability density function (PDF) [5]

$$f(x, y) = \begin{cases} x + cy, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Find the constant c that makes $f(x, y)$ a valid joint PDF.
- (ii) Find $P\left(0 \leq X \leq \frac{1}{2}, 0 \leq Y \leq \frac{1}{2}\right)$.
- (iii) Find the marginal density $h(y)$ and the conditional density $f(x|y)$.
- (iv) Find $P\left(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{2}\right)$.

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- Q.3(b) An insurance company has 25,000 automobile policy holders. If the yearly claim of a policy holder is a random variable with mean 320 and standard deviation 540, approximate the probability that the total yearly claim exceeds 8,300,000. Given that $\Phi(3.514) \approx 0.99978$. (Hint: use Central Limit Theorem.) [5]
- Q.4(a) A company manufactures metal rods and wants to estimate the average diameter of the rods produced. A random sample of 36 rods is taken, and the sample shows an average diameter of 2.6 millimeters. Assume that the population standard deviation is 0.3 millimeter. Find the 95% and 99% confidence intervals for the true average diameter of the rods. Given that $z_{0.025}=1.96$ and $z_{0.005}=2.575$. [5]
- Q.4(b) Consider a random sample x_1, x_2, \dots, x_n from a normal distribution $N(\mu, \sigma)$. Find the maximum likelihood estimators for μ and σ^2 . [5]
- Q.5(a) If S^2 is the variance of a random sample of size n then show that S^2 is an unbiased estimator of the parameter σ^2 . [5]
- Q.5(b) A real estate agent claims that 60% of all private residences being built today are 3 bedroom homes. To test this claim, a large sample of new residences is inspected; the proportion of these homes with 3 bedrooms is recorded and used as the test statistic. State the null and alternative hypotheses to be used in this test and determine the location of the critical region. [5]

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