

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(MID SEMESTER EXAMINATION)

CLASS: B Sc  
BRANCH: Mathematics and Computing

SEMESTER : I  
SESSION : MO/2025

SUBJECT: MA25117 REAL ANALYSIS AND MATRIX THEORY

TIME: 02 Hours

FULL MARKS: 25

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

- |   |     | CO | BL |
|---|-----|----|----|
| Q.1(a) Using Cauchy's criterion for convergence, prove that the sequence $\left\{1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}\right\}$ is convergent. | [2] | 1  | 3  |
| Q.1(b) Show that the sequence $\{s_n\}$ with $s_1 = 5$ and $s_{n+1} = \sqrt{4 + s_n}$ is convergent. Also find its limit.                                       | [3] | 2  | 2  |
| Q.2(a) Using Cauchy's theorem on limit, prove that : $\lim_{n \rightarrow \infty} \frac{n}{(n!)^{1/n}} = e$   | [2] | 1  | 3  |
| Q.2(b) Test the convergence of the series $x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \frac{5^5 x^5}{5!} + \dots$ , ( $x > 0$ )         | [3] | 2  | 4  |
| Q.3(a) Test the convergence of the series $\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \frac{1}{5^p} - \dots$ ( $p > 0$ )                   | [2] | 2  | 4  |
| Q.3(b) Show that the series $\sum \frac{\sin nx}{n^{3/2}}$ is absolutely convergent.  | [3] | 2  | 3  |
| Q.4(a) Is the sequence $\{g_n(x)\} = \frac{1 - nx^2}{(1 - nx^2)^2}$ ; $x \in \square$ point wise convergent? Explain.   | [2] | 2  | 2  |
| Q.4(b) Show that the sequence $\{g_n(x)\} = \frac{x}{1 + nx^2}$ ; $x \in \square$ is uniformly convergent.  | [3] | 2  | 3  |
| Q.5(a) Distinguish between point wise and uniform convergence of a sequence.  | [2] | 2  | 2  |
| Q.5(b) Test the convergence of the series $\sum \frac{[1 + \cos^2(nx)] \sin(nx)}{n^2}$ ; $x \in \square$  | [3] | 2  | 3  |