BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BSc | **SEMESTER : I**
BRANCH: MATHEMATICS AND COMPUTING | **SESSION : MO/2025**
SUBJECT: MA25117 REAL ANALYSIS AND MATRIX THEORY
TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Prove that the sequence $\{s_n\}$ defined by $s_{n+1}=\sqrt[3]{s_n+6}$, $s_1=1$ is convergent and find its limit.	[5]	1	3
Q.1(b)	Test the convergence of the series $\left(\frac{1}{3}\right)^2 x+\left(\frac{1.2}{3.5}\right)^2 x^2+\left(\frac{1.2.3}{3.5.7}\right)^2 x^3+\cdots\cdots$	[5]	1	2
Q.2(a)	Prove that the sequence $\left\{\frac{nx}{1+n^3x^2}\right\}$ converges uniformly to zero, where $x\in[0,1]$.	[5]	2	3
Q.2(b)	Prove that $\sum\frac{\sin\left(x^2+n^2x\right)}{n(n+1)}$ is uniformly convergent for all real values of x.	[5]	2	3
Q.3(a)	Determine whether the function $f(x)=\begin{cases}0, & when\ x\ is\ rational\\ 1, & when\ x\ is\ irrational\end{cases}$ is Riemann integrable in any real interval.	[5]	3	4
Q.3(b)	If a function $f(x)$ is continuous on $[a,b]$, then prove that there exists a number $\xi\in[a,b]$, such that $\int_a^b f\,dx=f(\xi)(b-a)$.	[5]	3	2
Q.4(a)	Find the vales of m and n for which the system of equations $x_1+mx_2+x_3=3$; $x_1+2x_2+2x_3=n$; $x_1+5x_2+3x_3=9$ are consistent. When will these equations have a unique solution?	[5]	4	3
Q.4(b)	Find the inverse of the following matrix A using LU decomposition method. $$\mathrm{A}=\begin{bmatrix}2&3&1\\1&2&3\\3&1&2\end{bmatrix}$$	[5]	4	3
Q.5(a)	Verify Cayley-Hamilton theorem for matrix A, and hence find A^{-1}, where $$\mathrm{A}=\begin{bmatrix}1&1&2\\1&3&-1\\-2&-1&1\end{bmatrix}$$	[5]	5	3
Q.5(b)	Examine whether the following matrix A is diagonalizable. If so, find the matrix P such that $\mathrm{P}^{-1}\mathrm{AP}$ is a diagonal matrix $$\mathrm{A}=\begin{bmatrix}5&-6&-6\\-1&4&2\\3&-6&-4\end{bmatrix}$$	[5]	5	4

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