BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: B.Sc. (H) **SEMESTER : Ist**
BRANCH: MATHEMATICS & COMPUTING **SESSION : MO/2025**

SUBJECT: MA25105 CALCULUS-I

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	If $y = \cos(msin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$	[5]		
Q.1(b)	Use Taylor's series expansion to expand $y = x^3 + 3x^2 + 2x + 1$ about the point $x = 1$.	[5]		
Q.2(a)	Find the asymptotes of the cubic equation $x^3 - 2y^3 + xy(2x-y) + y(x-y) + 1 = 0$.	[5]		
Q.2(b)	Define the curvature and radius of curvature of the curve. Find the radius of curvature of the conic $y - x = x^2 + 2xy + y^2$ at the origin.	[5]		
Q.3(a)	If $u = \frac{x+y}{1-xy}$ and $v = tan^{-1}x + tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so, find the relationship.	[5]		
Q.3(b)	Discuss the extreme values of the function $x^3 + y^3 - 3axy$.	[5]		
Q.4(a)	Find the reduction formula for the integral $\int sin^n x dx$ and hence deduce that $\int_0^{\frac{\pi}{2}} sin^n x dx = \begin{cases} \frac{(n-1)(n-3)(n-5)\ldots.2}{n(n-2)(n-4)\ldots.3}; if\ n\ is\ odd \\ \frac{(n-1)(n-3)(n-5)\ldots.2}{n(n-2)(n-4)\ldots.3}.\frac{\pi}{2}; if\ n\ is\ even \end{cases}$	[5]		
Q.4(b)	To prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.	[5]		
Q.5(a)	Find the area of the region bounded by the parabola $y^2 = 4ax$ and the straight line $y = x$.	[5]		
Q.5(b)	Find the volume of the solid enclosed by the planes $x = 0, y = 0, z = 0$ and tetrahedron $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.	[5]		

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