BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BTECH **SEMESTER : III**
BRANCH: CSE/AIML **SESSION : MO/2025**

SUBJECT: MA24205 DISCRETE MATHEMATICS

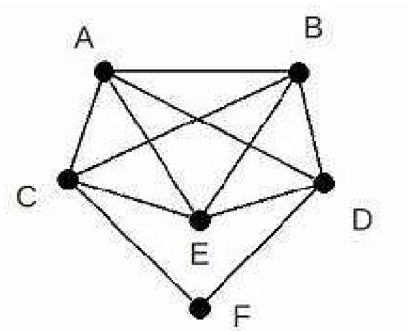
TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

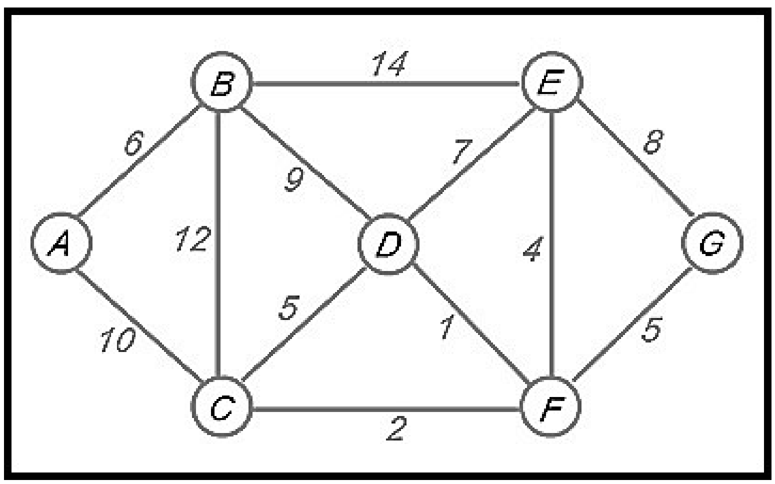
			CO	**BL**
Q.1(a)	When are two propositions p and q logically equivalent? Show using truth table that $\sim(p \to q)$ is logically equivalent to $p \wedge (\sim q)$.	[5]	1	3
Q.1(b)	Use mathematical Induction to prove that for integers $n \geq 2, (x+1)^n - nx - 1$ is divisible by x^2.	[5]	1	3
Q.2(a)	Solve for the particular solution of the recurrence relation $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2 - 2n + 1$.	[5]	2	3
Q.2(b)	Find the values of a, b, c in the recurrence relation $a_n = aa_{n-1} + ba_{n-2} + ca_{n-3}, n \geq 3$ if the general solution is $a_n = A(-1)^n + Bn(-1)^n + C3^n$, where A, B, C are arbitrary constants.	[5]	2	1
Q.3(a)	(i) Assume there are functions f and g such that $f(n)\ is\ O(g(n))$. Then $2^{(f(n))}$ is $O(2^{(g(n))})$. Prove or disprove the statement. (ii) Suppose f(n) = $n\sqrt{n}$ and $g(n) = n^2 - n$. Then $f(n) = O(g(n))$. If you think the answer is true, then find the corresponding c and k.	[5]	3	3
Q.3(b)	Given $A = \{1,2,3,4\}$ and R be the relation whose matrix is given by $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ Find the transitive closure of R using Warshall's algorithm and represent it in matrix form.	[5]	3	1
Q.4(a)	Let α, β be elements of the symmetric group S_4 given by $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ (i) Compute the permutations αβ, βα, and α^{-1} (give your answers in cycle notation). (ii) Write α, β, and $\alpha\beta\alpha^{-1}$ as a product of disjoint cycles. (iii) For each of the permutations α, β, and $\alpha\beta\alpha^{-1}$, state whether it is an odd or even permutation.	[5]	4	1
Q.4(b)	Consider a parity check matrix $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Find the $(3,6)$ group code $e_H : B^3 \to B^6$. Hence or otherwise, find the minimum distance of this group code.	[5]	4	1

PTO

Q.5(a) Define an Eulerian graph. Check if the given graph has an Euler circuit. Does it have a Hamiltonian circuit. Justify? [5] 5 1



Q.5(b) Find a minimal spanning tree for the given undirected graph using Prim's algorithm starting with vertex C. Is the graph an undirected tree? [5] 5 1



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