BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BTECH/IMSC **SEMESTER : I/ADD**
BRANCH: ALL/PHYSICS **SESSION : MO/2025**

SUBJECT: MA24101 / MA103 MATHEMATICS-I

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Test the convergence of the following series $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots$	[2]	1	2
Q.1(b)	Test the following series for convergence $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}-1}$	[3]	1	3
Q.1(c)	Verify that $f(x) = \frac{1}{x^2+1}, x \geq 1$ satisfies the condition of Cauchy's integral test and hence by using Cauchy's integral test determine whether $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges or diverges.	[5]	1	2
Q.2(a)	Test the consistency of the following system of linear equations using row operations: $\boldsymbol{x-4y+2z=6}$, $\boldsymbol{3x+2y-z=5}$, $\boldsymbol{2x-10y+5z=12}$.	[5]	2	3
Q.2(b)	Verify the Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, and hence determine the matrix $B = A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$, without explicitly performing full matrix, addition or subtraction (use Caley-Hamilton's reduction to simplify all higher powers of A).	[5]	2	2
Q.3(a)	Let $\mathbf{x} = \mathbf{u^2v + e^w}$, $\mathbf{y} = \mathbf{v^2w + \ln\ u (u > 0)}$, $\mathbf{z} = \mathbf{w^2u + uv}$. Find the Jacobian $\frac{\partial(\mathbf{x},\mathbf{y},\mathbf{z})}{\partial(\mathbf{u},\mathbf{v},\mathbf{w})}$. Also evaluate it at the point $(\mathbf{u},\mathbf{v},\mathbf{w}) = (\mathbf{1},\mathbf{1},\mathbf{0})$.	[5]	3	2
Q.3(b)	Examine the function $f(x,y) = x^2 + y^2 + xy - 6x - 6y + 14$ for extrema and find their values.	[5]	3	3
Q.4(a)	Change the order of integration of the following double integral $\int_0^1 \int_x^1 e^{y^2}\, dy\, dx$ and hence evaluate it.	[5]	4	2
Q.4(b)	Find the volume of the solid bounded by $z = 10 - 2x - y$ and situated in the first octant.	[5]	4	3
Q.5(a)	Find divergence of $\vec{F}$, where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$. Determine whether $\vec{F}$ is a conservative vector field or not?	[5]	5	2
Q.5(b)	Apply Green's theorem to evaluate the integral $\int_C (x^5 + 3y)dx + (2x - e^{y^3})dy$, where C is the circle $(x-1)^2 + (y-5)^2 = 4$.	[5]	5	3

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