BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: B.TECH/IMSC **SEMESTER : I/ADD**
BRANCH: ALL **SESSION : MO/2025**

SUBJECT: MA24101 MATHEMATICS - I

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

			CO	BL
Q.1(a)	Test for the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\cdots\infty$.	[2]	1	2
Q.1(b)	Examine the sequence $\boldsymbol{a_n=\frac{2n+1}{n+1}}$ is monotonic, bounded and convergent.	[3]	1	2
Q.2(a)	Test the convergence of the series $\sum_{n=1}^{\infty}5^{-n-(-1)^n}$.	[2]	1	2
Q.2(b)	Prove that the series $$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$$ is absolutely convergent for $-1<x<1$ but conditionally convergent for $x=1$.	[3]	1	2
Q.3(a)	Find the rank of the matrix: $\begin{pmatrix}1&2&3\\1&4&2\\2&6&5\end{pmatrix}$.	[2]	2	1
Q.3(b)	Determine the values of a and b for which the system of linear equations $$x+y+z=3,$$ $$x+y+2az=7,$$ $$x+2y+3az=b,$$ has (a) no solution, (b) a unique solution, and (c) infinitely many solutions.	[3]	2	1
Q.4(a)	Examine the linear dependence or linear independence for the system of vectors (3,1,-4), (2,2,-3) and (0,-4,1).	[2]	2	2
Q.4(b)	Find the eigenvalues and the corresponding eigenvectors for the given matrix $\begin{bmatrix}2&0&-1\\0&\frac{1}{2}&0\\1&0&4\end{bmatrix}$.	[3]	2	1
Q.5(a)	Let $x^x y^y z^z = c$, where c be a constant and z is a function of two independent variables x and y. Prove that at z = y = x; $\frac{\partial^2 z}{\partial x \partial y}=-(x\ln ex)^{-1}$.	[2]	3	2
Q.5(b)	Let $u=\sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{-\sin u\cos 2u}{4\cos^3 u}$.	[3]	3	2

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