

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/2025)

CLASS: IMSC
BRANCH: MATHEMATICS AND COMPUTING

SEMESTER : III
SESSION : MO/2025

SUBJECT: MA202R1 ABSTRACT ALGEBRA

TIME: 02 HOURS

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates
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|--------|--|-----|----|
| Q.1(a) | Use Euclidean Algorithm to compute GCD (12378, 3054). | [2] | 1 |
| Q.1(b) | Let '+' be defined on the set $Z_2 \times Z_2$ by $(a, b) + (c, d) = (a+_2c, b+_2d)$ for $(a, b), (c, d) \in Z_2 \times Z_2$, where $+_2 = \text{addition (mod 2)}$. Show that $Z_2 \times Z_2$ is a abelian group under addition. | [3] | 1 |
| Q.2(a) | Find order of each element of $(Z_6, +)$. | [2] | 1 |
| Q.2(b) | Show that the relation ρ on the set $S = Z$ is an equivalence relation, where $\rho = \{(a, b) \in Z \times Z : 3a + 4b \text{ is divisible by } 7\}$. by | [3] | 1 |
| Q.3(a) | Let $G = (Z, +)$ and $\phi: G \rightarrow G$ is defined by $\phi(x) = x + 3, x \in Z$. Prove that ϕ is a homomorphism. | [2] | 2 |
| Q.3(b) | Prove that, if $\phi: (G, \cdot) \rightarrow (G', *)$ be a homomorphism, then $\ker \phi$ is subgroup of G and $Im \phi$ is a subgroup of G' . | [3] | 2 |
| Q.4(a) | Define Quotient group. If $G = (Z_6, +)$ and $H = \{\bar{1}, \bar{4}\}$. Find G/H . | [2] | 2 |
| Q.4(b) | Find $[G:H]$, where $H = \{\rho_0, \rho_1, \rho_2\}$ and $G = (S_3, *)$, where $\rho_0 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \rho_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \rho_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$ | [3] | 2 |
| Q.5(a) | Find the conjugacy classes in the group $G = \{1, -1, i, -i\}$ (group w.r.to multiplication). | [5] | 3 |

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