BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc — **SEMESTER : III**
BRANCH: MATHEMATICS AND COMPUTING — **SESSION : MO/2025**

SUBJECT: MA202R1 ABSTRACT ALGEBRA

TIME: 3 Hours — **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Show the binary composition table defined in the set $G = \{\pm1, \pm i, \pm j, \pm k\}$, where $i^2 = j^2 = k^2 = -1$ and $ij = -ji = k, jk = -kj = i, ki = -ik = j$.	[5]	1	
Q.1(b)	Define cyclic groups. Find all the cyclic subgroups of the group $(Z_5 , +)$.	[5]	1	
Q.2(a)	Define normal subgroups. Prove that the improper subgroup $H = G$ is a normal subgroup of $(G, .)$.	[5]	2	
Q.2(b)	Let $\phi: (G, .) \to (G', *)$ be homomorphism. Then prove that ϕ is one to one iff $ker\, \phi = \{e_G\}$.	[5]	2	
Q.3(a)	Let G be a finite group with $\mid G \mid = 56$. Show that 1. G has a subgroup of order 8 and a subgroup of order 7. 2. Determine the possible number of Sylow-7 subgroups.	[5]	3	
Q.3(b)	Find elementary divisors and invariant factors for a group of order 50.	[5]	3	
Q.4(a)	Prove that the characteristic of an integral domain is either zero or a prime no.	[5]	4	
Q.4(b)	Define Prime ideal. Prove that the ideal $(2Z, +, .)$ in the ring $(Z, +, .)$ is a prime ideal.	[5]	4	
Q.5(a)	Determine whether the map $f: \mathbb{C} \to \mathbb{R}$ given by $f(a + bi) = a - bi$ is a ring homomorphism.	[5]	5	
Q.5(b)	Define irreducible elements in an integral domain. Determine if 2 is irreducible in $\mathbb{Z}[i]$.	[5]	5	

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