BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc **SEMESTER : III**
BRANCH: Mathematics **SESSION : MO/2025**

SUBJECT: MA201R1 PARTIAL DIFFERENTIAL EQUATIONS

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Find the general integral of the PDE: $x^2p + y^2q=(x + y)z$.	[5]	1	3
Q.1(b)	Find the integral surface of the equation $yp + xq - z = 0$, which passes through the curve $z=x^3$, $y=0$.	[5]	1	3
Q.2(a)	Use Charpit's method to find the complete integral of $(p^2 + q^2)y = qz$.	[5]	2	3
Q.2(b)	Transform the following PDE into the canonical form: $u_{xx} + 4u_{xy} + u_x = 0$.	[5]	2	4
Q.3(a)	Use D'Alembert's formula to solve: $u_{tt}=4u_{xx}$, $(-\infty < x < \infty, t>0)$ with $u(x,0) = \cos 3x$, $u_t(x,0) = \sin^2 x$. (without derivation)	[5]	3	3
Q.3(b)	Derive the solution of the wave equation $u_{tt}=5u_{xx}$, $(0 < x < 1, t> 0)$, $u(0,t) = 0$, $u(1,t) = 0$, $(t>0)$, $u(x,0) = \sin(\pi x) + 3\sin(5\pi x)$, $u_t(x,0) = \sin(3\pi x) - 13\sin(7\pi x)$, $(0 \le x \le 1)$.	[5]	3	4
Q.4(a)	Solve the following heat equation in a finite rod $u_t=c^2u_{xx}$, $(0 < x < \pi, t> 0)$, $u(0,t) = u(\pi,t) = 0$, $(t> 0)$, $u(x,0) = 3\sin x - 5\sin 3x$, $(0 \le x \le \pi)$.	[5]	4	3
Q.4(b)	Use the general solution derived in the above problem to solve $u_t=4u_{xx}$, $(0 < x < 1, t> 0)$, $u(0,t) = 1$, $u(1,t) = 2$, $(t> 0)$, $u(x,0) = \sin(\pi x) - \sin(3\pi x)$, $(0 \le x \le 1)$.	[5]	4	4
Q.5(a)	Solve the Dirichlet's BVP $u_{xx} + u_{yy} = 0$, $(0 < x < 1, 0 < y < 1)$, $u(x,0) = 0$, $u(x,1) = 0$, $(0 \le x \le 1)$, $u(0,y) = 0$, $u(1,y) = \sin(\pi y)$, $(0 \le y \le 1)$.	[5]	5	4
Q.5(b)	Find the solution of the Laplace equation $u_{xx} + u_{yy} = 0$, $(0 < x < \pi, 0 < y < \pi)$, $u_y(x,0) = \cos x$, $u_y(x,\pi) = 0$, $(0 \le x \le \pi)$, $u_x(0,y) = 0$, $u_x(\pi,y) = 0$, $(0 \le y \le \pi)$.	[5]	5	4

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