BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : I**
BRANCH: Mathematics & Computing **SESSION : MO/2025**

SUBJECT: MA109 MATRIX THEORY

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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			CO	BL
Q.1(a)	If P and $\boldsymbol{Q}$ are Hermitian matrices, then show that $\mathbf{PQ}-\boldsymbol{QP}$ is skew-Hermitian.	[2]	CO 1	2
Q.1(b)	If $\boldsymbol{A}$ is a skew-Hermitian matrix, then show that $\boldsymbol{iA}$ is Hermitian.	[3]	CO 1	2
Q.2(a)	Let $\boldsymbol{A}$ be an $\boldsymbol{n}\times\boldsymbol{n}$ idempotent matrix (i.e., $A^2 = A$). Prove that $\boldsymbol{I}-\boldsymbol{A}$ is also idempotent, where $\boldsymbol{I}$ is the $\boldsymbol{n}\times\boldsymbol{n}$ identity matrix.	[2]	CO 1	2
Q.2(b)	Let $\boldsymbol{A}$ be an $\boldsymbol{n}\times\boldsymbol{n}$ real matrix. Prove that $\boldsymbol{A}$ is an orthogonal matrix if and only if its transpose $\boldsymbol{A^T}$ is equal to its inverse $\boldsymbol{A^{-1}}$. Additionally, show that for an orthogonal matrix $\boldsymbol{A}$, the determinant satisfies: $\mathbf{det}\,(\boldsymbol{A}) = \pm\mathbf{1}$.	[3]	CO 1	2
Q.3(a)	Using row echelon method, find the rank of the matrix: $\begin{pmatrix} \mathbf{0} & \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} & \mathbf{5} \end{pmatrix}$.	[2]	CO 1	2
Q.3(b)	Test the consistency of the system of equations: $\boldsymbol{x-y+z=3;\ x+y-z=5;\ x+2y-3z=7}$.	[3]	CO 2	3
Q.4	Find the value of λ for which the system of equations is consistent: $\boldsymbol{x+y+z=1;\ x+2y-2z=1;\ \lambda x+y+z=1}$. Also, find its unique solution.	[5]	CO 2	3
Q.5	Apply the Gauss-elimination method to solve the system of equations: $\boldsymbol{x+y+z=6;\quad 2x-y+3z=9;\quad 3x+4y-z=8}$.	[5]	CO 3	3

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