BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER: I**
BRANCH: MATHEMATICS & COMPUTING **SESSION: MO/2025**

SUBJECT: MA102 REAL ANALYSIS

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Find the supremum and infimum of the following two set. Also, check that the sets are bounded or not? $\boldsymbol{A} = \{\boldsymbol{x} \in \boldsymbol{R}: \mid \boldsymbol{x} - \boldsymbol{3} \mid < \boldsymbol{2}\}$, $\boldsymbol{B} = \left\{\frac{\boldsymbol{m+n}}{\boldsymbol{mn}} : \boldsymbol{m}, \boldsymbol{n} \in \mathbb{N}\right\}$	[5]	CO1	2
Q.1(b)	For the set $S = \left\{(-1)^n \left(1 + \frac{1}{n}\right) : n \in \mathbb{N}\right\}$, determine its interior points, check whether it is open, find all its limit points, and decide whether it is closed.	[5]	CO1	2
Q.2(a)	Prove that every convergent sequence is bounded, but every bounded sequence is not convergent.	[5]	CO2	2
Q.2(b)	Use sandwich theorem to show that: $$\lim_{n\to\infty}\left(\frac{\boldsymbol{1}}{\sqrt{\boldsymbol{n^2+1}}} + \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{n^2+2}}} + \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{n^2+3}}} + \cdots + \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{n^2+n}}}\right) = \boldsymbol{1}$$	[5]	CO2	3
Q.3(a)	Using Cauchy Integral test, examine whether the series $$\sum_{\boldsymbol{n=2}}^{\infty} \frac{\boldsymbol{1}}{\boldsymbol{n(\log n)^2}}$$ is convergent or divergent.	[5]	CO3	3
Q.3(b)	Check whether the given series is absolutely or conditionally convergent. $$\sum_{\boldsymbol{n=1}}^{\infty} \frac{\boldsymbol{(-1)^{(n+1)}}}{\sqrt{\boldsymbol{n}}}$$	[5]	CO3	3
Q.4(a)	For each $n \in N$, let $\boldsymbol{f_n(x)} = \boldsymbol{1} - \frac{\boldsymbol{x^n}}{\boldsymbol{n}}$ $\forall\, \boldsymbol{x} \in [\boldsymbol{0}, \boldsymbol{1}]$, then check using the Mn-test the sequence of function $\{f_n\}$ is uniformly convergence or not over $[0, 1]$.	[5]	CO4	2
Q.4(b)	Show that the series of function $\sum \frac{1}{n^3+n^4x^2}$ is uniformly convergent for all real x. (Hint: Use Weierstrass' M-test).	[5]	CO4	3
Q.5(a)	A function f(x) is defined on [0,1] by, $$\boldsymbol{f(x)} = \begin{cases} \boldsymbol{1}, & \textbf{if } \boldsymbol{x} \textbf{ is rational} \\ \boldsymbol{0}, & \textbf{if } \boldsymbol{x} \textbf{ is irrational}. \end{cases}$$ Examine the function f(x) is Reimann integrable or not on [0,1]?	[5]	CO5	2
Q.5(b)	Let $\boldsymbol{f}: [\boldsymbol{a}, \boldsymbol{b}] \to \mathbb{R}$ be bounded on [a, b] and $\boldsymbol{P}$ be any partition of [a, b]. Prove that $\boldsymbol{L(P, f)} \leq \boldsymbol{U(P, f)}$.	[5]	CO5	2

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