

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)**

CLASS: IMSC
BRANCH: MATHS & COMP.

SEMESTER : I
SESSION : MO/2025

SUBJECT: MA101 CALCULUS - I

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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|--|----------|-------|
| Q.1(a) Show that the function $f(x) = x^2 - 6x + 1$ satisfies Lagrange's Mean Value Theorem. Also, find the coordinates of a point at which the tangent to the curve represented by the above function is parallel to the chord joining A (1, - 4) and B (3, - 8). | [5] | 1 1.5 |
| Q.1(b) Prove that the curve $y = az^3 + bx^2 + cx + d$ always has one and only one point of inflection. | [5] | 1 1.5 |
| Q.2 Show that the radius of curvature at any point on the curve $x = ae^{\theta} (\sin \theta - \cos \theta)$, $y = ae^{\theta} (\sin \theta + \cos \theta)$ is twice the perpendicular distance from the tangent at the point from the origin. | [10] | 2 2 |
| Q.3 Find the necessary Lagrange's conditions and the minimum values for the function $xy + 2xz + 2yz$ subject to $xyz = 32$. | S = [10] | 3 2.5 |
| Q.4(a) Evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2} dx$ using differentiation under the integral sign and hence, show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$. | [5] | 4 2 |
| Q.4(b) Show that $\int_0^{\frac{\pi}{4}} (\sqrt{\tan \theta} + \sqrt{\sec \theta}) d\theta = \frac{1}{2} \Gamma\left(\frac{1}{4}\right) \left[\Gamma\left(\frac{3}{4}\right) + \frac{\sqrt{\pi}}{\Gamma\left(\frac{3}{4}\right)} \right]$. | [5] | 4 2 |
| Q.5 For the given figure, find the volume of the solid generated when the curve is revolved about the x - axis. Also determine the surface area of the solid formed by this revolution when $a = 4$ and $b = 2$. | [10] | 5 2.5 |

