

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)**

**CLASS:** Integrated Master of Science  
**BRANCH:** Quantitative Economics and Data Science

**SEMESTER :** IX  
**SESSION :** MO/2025

**SUBJECT: ED539 SOCIAL AND ECONOMIC NETWORKS: THEORY AND APPLICATION**

**TIME:** 3 Hours

**FULL MARKS:** 50

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Before attempting the question paper, be sure that you have got the correct question paper.
  5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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- |   | [4] | [6] | [6] | [4] | [2+2] | [6] |
|---|-----|-----|-----|-----|-------|-----|
| Q.1(a) Consider a random graph $G(n, p)$ where $n$ is the total number of nodes and $p$ is the probability of having an edge between any two nodes in the graph. Derive the expression for the probability of generating a graph with $m$ edges.  | CO1 |     |     | BL  |       | 4   |
| Q.1(b) Prove that for large $n$ and small $p$ in $G(n, p)$ , the expression can be approximated by Poisson distribution, where the parameter $\lambda$ associated with Poisson distribution is given by the relation: $\lambda = n.p$ .   | CO1 |     |     |     |       | 5   |
| Q.2(a) Consider a game where each player $i$ chooses action $x_i \in \{0,1\}$ . Every player chooses action 1 if and only if none or all of his/her neighbors choose it. If none of the neighbors chooses it, then there is a cost $c; 0 \leq c \leq 1$ , for choosing action 1. If all neighbors choose it, there is no cost to pay (maximum pay-off). Among the following network states (I), (II) and (III), which do you find the best in terms of total cost incurred by all the nodes in the network? | CO3 |     |     |     |       | 3   |
|   |     |     |     |     |       |     |
| Q.2(b) Define complete lattice in the context of games on networks with a proper example.   | CO3 |     |     |     |       | 2   |
| Q.3(a) Assume that there is a new technology in the market. Let the fraction of the population who adopted the new technology is $g(t)$ . Let, $m$ be the rate of spontaneous adoption and $n$ be the rate of imitation of adoption. Following the BASS model, write down the characteristic equation using the above information. Discuss about two extreme conditions: $g(t) = 0$ and $g(t) = 1$ .  | CO2 |     |     |     |       | 3   |
| Q.3(b) Consider a SIS (Susceptible-Infected-Susceptible) model. Let, the percentage of people infected be $p$ and the rate of getting well be $\delta > 0$ . The probability of getting infected is proportional to number of infected neighbors with rate $\nu > 0$ . Let, the number of spontaneously infected people be $\phi$ . Considering steady state, formulate and solve the characteristic equation. Discuss about the solutions when there is no spontaneous infection.                          | CO2 |     |     |     |       | 3   |

- Q.4(a) Consider a network  $G = (V, E)$  with set of vertices  $V = \{v_1, v_2, \dots, v_n\}$  and  $K$  communities. The probability of belongingness of each vertex into  $k$ -th community ( $1 \leq k \leq K$ ) is  $\pi_k$ . Find the maximum likelihood estimate under known group assignment. [5] CO3 4,5
- Q.4(b) (I) Write down the expression for the marginal likelihood when community assignments are unobserved. What is the problem with the marginal likelihood expression? [2+3] CO3 4  
 (II) Write down the E-step and M-step of EM algorithm to find the maximum likelihood estimates of the associated parameters.
- Q.5(a) Consider a network  $G = (V, E)$  where the set of vertices is given by  $V = \{v_1, v_2, \dots, v_n\}$ .  $G$  has  $K$  inherent communities and each vertex  $v_i$ ;  $1 \leq i \leq n$  can belong to exactly one community. Given the above information, formulate the objective function that is best suited to capture the community structure. [6] CO4 4,5
- Q.5(b) In the context of social network structure, define the following with examples: [2+2] CO4 2  
 (I) Triadic closure  
 (II) Neighborhood overlap.

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