

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)**

**CLASS: INTEGRATED MASTER OF SCIENCE  
BRANCH: QUANTITATIVE ECONOMICS AND DATA SCIENCE**

**SEMESTER : IX  
SESSION : MO/2025**

**SUBJECT: ED501 DESIGN AND ANALYSIS OF ALGORITHMS**

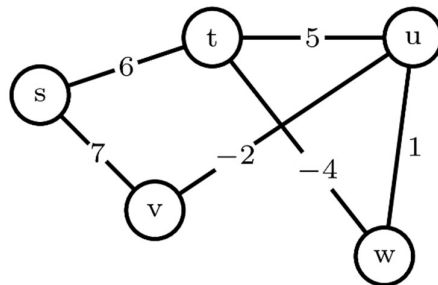
**TIME: 3 Hours**

**FULL MARKS: 50**

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

- |        |   |              |            |
|--------|---|--------------|------------|
|        |   | <b>CO</b>    | <b>BL</b>  |
| Q.1(a) | (I) Using the definitions of asymptotic notations, prove that if $f(n) = O(h(n))$ and $g(n) = O(h(n))$ , then $f(n) + g(n) = O(h(n))$ .<br>(II) Let $f(n) = a_0 \cdot n^d + a_1 \cdot n^{d-1} + \dots + a_n$ , where $a_0 \neq 0$ and each $a_i; 0 \leq i \leq n$ may be positive or negative. Prove that $f(n) = O(n^d)$ . | [2+3]<br>CO1 | 4,5<br>4,5 |
| Q.1(b) | (I) Solve the following recurrence relation: $T(n) = \begin{cases} T(n-1) + 2n; & n > 1 \\ O(1); & n = 1 \end{cases}$<br><br>(II) Solve the following recurrence relation using recursion tree method:<br>$T(n) = \begin{cases} T(n/2) + c \cdot n; & n > 1 \\ O(1); & n = 1 \end{cases}$                                   | [2+3]<br>CO1 | 4,5<br>4,5 |
| Q.2(a) | Prove that, if $n$ integers are used to form an almost complete binary tree (ACBT) and then heap properties are utilized to form a heap structure (either min-heap or max-heap), the time complexity of building heap will be $O(n)$ .  | [5]<br>CO2   | 5<br>5     |
| Q.2(b) | Apply Merge Sort algorithm to sort the list of elements: 14, 2, 8, 5, 14, 5.<br><br>Check whether Merge Sort is stable in the following two cases: (I) during element-wise comparison while merging two sub-arrays ' $\leq$ ' is used; (II) during element-wise comparison ' $<$ ' is used.                                 | [3+2]<br>CO2 | 3,5<br>3,5 |
| Q.3(a) | Consider the following graph $G = (V, E)$ where the set of vertices $V = \{s, t, u, v, w\}$ and $s$ is the source vertex.   | [2+4]<br>CO3 | 2,3<br>2,3 |



- (I) Which single-source shortest path finding algorithm you will choose to find the shortest paths? Justify your answer.
- (II) Using the same algorithm, find the shortest path to all vertices  $t, u, v, w$  from the source vertex  $s$ .
- |        |  |            |        |
|--------|--|------------|--------|
| Q.3(b) | Given any arbitrary graph $G = (V, E)$ . Let $T$ be a breadth-first search tree, let $x$ and $y$ be vertices in $T$ belonging to layers $L_i$ and $L_j$ respectively, and let $(x, y)$ be an edge of $G$ . Prove that $i$ and $j$ differ by at most 1. | [4]<br>CO3 | 5<br>5 |
|--------|--|------------|--------|

- Q.4(a) Consider two  $n$  digit numbers  $A = a_1a_2 \dots a_n$  and  $B = b_1b_2 \dots b_n$ . Discuss the strategy behind Karatsuba's approach for integer multiplication. Formulate the recurrence relation and find the time complexity. [5] CO4 2
- Q.4(b) (I) Consider two  $n \times n$  matrices  $A$  and  $B$ . Using recurrence relations, show that Strassen's approach is better than standard matrix multiplication in terms of time complexity. [3+2] CO4 2

(II) What are the limitations of Strassen's matrix multiplication approach?

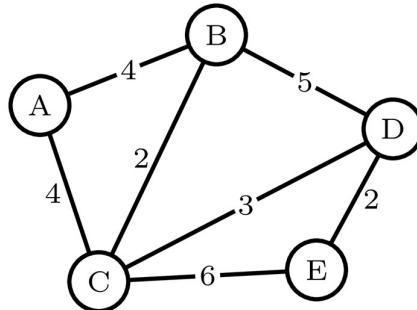
- Q.5(a) (I) A thief has a knapsack that has a weight capacity of 15 kg. The available items are listed as follows: [3+2] CO5 2,3

Item	Profit (Rs.)	Weight (kg)
$Ob_1$	36	6
$Ob_2$	60	10
$Ob_3$	24	4
$Ob_4$	20	5

Use a greedy approach to determine which items or fraction of items should be selected to maximize total profit. Find the profit.

(II) Consider a 0/1-Knapsack problem. Let, the knapsack capacity be  $m$  kg and total number of available items be  $n$ . Let, the weights of  $n$  items be  $w_1, w_2, \dots, w_n$  and the profits from items be  $p_1, p_2, \dots, p_n$ . Write down the recurrence relation for solving the problem.

- Q.5(b) Consider the graph  $G = (V, E)$  where the set of vertices  $V = \{A, B, C, D, E\}$ . [5] CO5 3



Find the minimum cost spanning tree using Union-Find operations of disjoint set data structures. Find the minimum cost of spanning the set of vertices  $V$ .

:::::19/11/2025:::::E