

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)

CLASS: IMSc  
BRANCH: QEDS

SEMESTER : VII  
SESSION : MO/2025

SUBJECT: ED411 ADVANCED MICROECONOMICS

TIME: 3 Hours

FULL MARKS: 50

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|----|--------|
| <p>Q.1 A policy analyst is studying how urban households allocate their spending between:</p> <ul style="list-style-type: none"> <li>• X: units of public-transport rides, and</li> <li>• Y: units of app-based mobility services (ride-hailing).</li> </ul> <p>Because households value ride-hailing disproportionately when public-transport reliability falls, their preferences are modeled using:</p> $U(X, Y) = \ln(X + 2Y).$ <p>Let the prices of the two transport modes be <math>p_X</math> and <math>p_Y</math>, and let income be <math>M</math>.</p> <p>(a) Derive the expenditure function <math>E(p_X, p_Y, U)</math> for this utility function.</p> <p>(b) Using your expression for expenditure, derive the indirect utility function <math>V(p_X, p_Y, M)</math>. Then verify that the expenditure function can be obtained by <i>inverting</i> this indirect utility function, thereby confirming the duality relationship.</p> <p>(c) From the expenditure function, derive the Hicksian (compensated) demand functions for <math>X</math> and <math>Y</math>.</p> | [3+4+3] | 1  | 1<br>5 |
| <p>Q.2 A young entrepreneur has a constant absolute risk-averse utility function given by <math>U(w) = 1 - e^{-0.004w}</math>, where <math>w</math> is her final wealth. Her aunt gifts her a startup grant of ₹80,000. She decides to invest ₹20,000 of this amount in a risky tech prototype. The prototype's return is uncertain and yields:</p> <ul style="list-style-type: none"> <li>• ₹0 with probability 0.25,</li> <li>• ₹30,000 with probability 0.35,</li> <li>• ₹60,000 with probability 0.40.</li> </ul> <p>Compute her final wealth level in each possible outcome and the expected utility from this risky investment.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                             | [5+5]   | 2  | 3<br>4 |
| <p>Q.3(a) Consider a market with demand <math>P = 90 - Q</math>. The incumbent firm has constant average and marginal cost of Rs.20. A potential entrant has constant average cost of Rs.25. If the entrant enters, it expects to capture <b>40%</b> of the total market demand and also requires the payment of a <b>fixed cost</b> of ₹200.</p> <p>a. Derive the zero-profit (break-even) condition for the entrant.</p> <p>b. Calculate the <b>limit price</b>, i.e., the highest price the incumbent can charge without inducing entry.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | [2+3+5] | 3  | 3<br>4 |
| <p>Q.3(b) Two identical firms, <b>A</b> and <b>B</b>, are located at opposite points on a circular city of circumference 1 (so each half of the circle is 0.5 units long). Consumers are uniformly distributed with density 1 per unit distance and buy one unit each. Transportation cost per unit distance is <math>t=10</math>. Both firms have marginal cost, <math>c=20</math>. Derive the symmetric Nash equilibrium price. Two identical firms, <b>A</b> and <b>B</b>, are located at opposite points on a circular city of circumference 1 (so each half of the circle is 0.5 units long). Consumers are uniformly distributed with density 1 per unit distance and buy one unit each. Transportation cost per unit distance is <math>t=10</math>. Both firms have marginal cost, <math>c=20</math>. Derive the symmetric Nash equilibrium price.</p>                                                                                                                                                                                                                         |         |    |        |

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- Q.4 A logistics company, SwiftTrack Corp., hires freelance delivery agents who can be of two unobservable types: Efficient (E) or Inefficient (I). The company must assign delivery intensity levels  $d \in \{d_H, d_L\}$ . Each agent incurs a private cost of effort described by a non-linear type-dependent cost function:

$$C_\theta(d) = \begin{cases} ad^2 - bd & \text{for type } \theta = E \\ ad^2 & \text{for type } \theta = I \end{cases}$$

where  $a > 0$ ,  $b > 0$ , and the firm earns revenue  $R(d) = \sqrt{d}$  from delivery intensity  $d$ .

- i. Construct a (detailed and complete) screening model of adverse selection, and derive the first-best contract menu  $(d_\theta^{FB}, w_\theta^{FB})$  that the firm would offer if types were observable.
- ii. Show formally why the first-best menu becomes incentive-incompatible when agent type is private information.

- Q.5(a) “Every competitive equilibrium is Pareto efficient.” Explain and justify this statement. [5+5] 5 4
- Q.5(b) First Welfare Theorem fails to ensure a just or equitable outcome. Justify. 4

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