BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC/PRE-PHD | **SEMESTER: VII/I**
BRANCH: CQEDS | **SESSION: MO/2025**

SUBJECT: ED403 LARGE SAMPLE THEORY

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions, each of 10 marks, and a total of 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data handbook/Graph paper, etc., to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Suppose X follows an exponential distribution with location parameter θ and scale parameter λ =1. The probability density function of X is given by, f (x, θ) = exp {−(x − θ)}, x ≥ θ. Show that $X_{(1)}$ is a biased but consistent estimator for the location parameter.	[7]	1	2
Q.1(b)	State the Weak Law of Large Numbers and the Central Limit Theorem with their assumptions.	[3]	1	3
Q.2(a)	Explain the variance stabilizing technique with an example.	[5]	2	2
Q.2(b)	Suppose X1, X2, ..., Xn is a random sample from a Bernoulli B (1, θ). Find a CAN estimator of θ (1 − θ) when θ ∈ (0, 1) − {1/2}, that is, θ cannot take the value ½.	[5]	2	3
Q.3(a)	Suppose X1, X2, ..., Xn are independent and identically distributed random variables with the following probability mass function. P [X_1 = 1] =(1−θ_1)/2, P [X_1 = 2] = θ_2/2, P [X_1 = 3] = θ_1/2 and P [X_1 =4] = (1−θ_2)/2. Derive a likelihood ratio test procedure to test H_0: θ_1 = 1/2, θ_2 = 1/4 against a sided alternative hypothesis.	[8]	3	4
Q.3(b)	Consider the regularized linear regression model $$\hat{\beta}_n = \arg\min_{\beta} \{\frac{1}{n}\sum_{i=1}^{n} (Y_i - X_i^{\top}\beta)^2 + \lambda_n \parallel \beta \parallel^2\},$$ where $\lambda_n > 0$ depend on the sample size n. What conditions on λ_n ensure that $\hat{\beta}_n$ is a consistent estimator of the true parameter β_0? Just give the condition; no need to prove those conditions.	[2]	3	3
Q.4	Let X1, ..., Xn, be an i.i.d. random sample from a Poisson distribution, that is, Poi(λ). Prove the existence and uniqueness of MLE and obtain the Fisher information for the parameter λ.	[10]	4	3
Q.5(a)	Define U statistics. Also, discuss the asymptotic properties of U-statistics.	[6]	5	4
Q.5(b)	State continuous mapping and Slutsky's theorems.	[4]	5	4

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