BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION MO/2025)

CLASS: IMSC/PRE-PHD | **SEMESTER : VII/I**
BRANCH: QEDS | **SESSION : MO/2025**

SUBJECT: ED401 ADVANCED ANALYSIS

TIME: 03 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates.
5. All the notations used in the question paper have usual meanings.

	Question	Marks	CO	BL
Q.1(a)	Define a separable metric space and a complete metric space. Show that the set of rational numbers $\mathbb{Q}$, under the usual metric $d(x,y) = \|x - y\|$, where $x, y \in \mathbb{Q}$ is separable in $\mathbb{R}$, but not complete.	[2+3]	CO1	1,2
Q.1(b)	Consider the data points $x_0 = (3,2), x_1 = (2,2), x_2 = (2,3)$. Compute the distance from x_0 to x_1 and x_2 with respect to Euclidean metric and sup metric. Find the nearest neighbour of x_0 under each metric.	[4+1]	CO1	2
Q.2(a)	Show that every finite dimensional subspace Y of a normed space $(X, \|\|.\|\|)$ is complete in X.	[5]	CO2	3
Q.2(b)	Let $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \in \mathbb{R}_{2\times2}$. Compute $\|\|A\|\|_1$ (maximum absolute row sum norm) and $\|\|A\|\|_2$ (spectral norm). Illustrates the norm equivalency of these norms in $\mathbb{R}_{2\times2}$.	[4+1]	CO2	3
Q.3(a)	Define bounded linear operator. Let $T: C[0,1] \to \mathbb{R}$ be defined by $T(f) = \int_0^1 (x+2) f(x)\, dx$. Show that T is bounded linear functional and find $\|\|T\|\|$.	[5]	CO3	4
Q.3(b)	Define Hilbert space and give an example of a Hilbert space. Give an example of a normed space which is not an inner product space. Justify your answer.	[5]	CO3	4
Q.4(a)	Let $x = (2,3) \in \mathbb{R}^2, M = span\{(1,2)\}$. Find the projection of x onto M using standard inner product defined on $\mathbb{R}^2$. Hence, find the error vector.	[3]	CO4	5
Q.4(b)	Use Gram-Schimdt process to find an orthonormal basis for all the polynomials of degree atmost 2 $\mathcal{P}^2[-1,1]$ with respect to the inner product $< f, g > = \int_{-1}^{1} f(x) g(x)\, dx$.	[7]	CO4	5
Q.5(a)	Define Hilbert adjoint operator. Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear operator represented by a matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$. Find adjoint operator T^* with respect to the inner product $\langle x, y \rangle = x^T y$.	[4]	CO5	5
Q.5(b)	State and prove Hahn Banach theorem on normed space.	[6]	CO5	5

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