

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)

CLASS: IMSc  
BRANCH: QEDS

SEMESTER : V  
SESSION : MO/2025

SUBJECT: ED307 PARAMETRIC INFERENCE

TIME: 3 Hours

FULL MARKS: 50

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Before attempting the question paper, be sure that you have got the correct question paper.
  5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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- |   | CO          | BL         |
|---|-------------|------------|
| Q.1 State the Cramer Rao Inequality. Let $X_1, X_2, \dots, X_n$ be a random sample from Poisson distribution with parameter $\lambda$ . Find the mle of $\lambda$ . Obtain the Cramer Rao lower bound to the variance of an unbiased estimator for $\lambda$ . Hence, find the UMVUE for $\lambda$ .  | [10]<br>C01 | II,<br>III |
| Q.2 State and prove Rao Blackwell theorem. Let $X_1, X_2, \dots, X_n$ be random sample from $f(x, \theta) = \theta^x(1 - \theta)^{1-x}$ for $x=0,1$ . Let $T = X_1$ be an unbiased estimator of $\theta$ and $S = \sum_{i=1}^n X_i$ be sufficient for $\theta$ . Prove the Rao Blackwell theorem in this case.  | [10]<br>C02 | II,<br>III |
| Q.3 What do you mean by squared error loss function. Show that posterior mean is the Bayes estimate under squared error loss function.<br>Let $X_1, X_2, \dots, X_n$ be a random sample from Poisson distribution with parameter $\lambda$ . Let $\lambda$ is distributed as gamma with parameters $\alpha$ and $\beta$ . Find the Bayes estimate under squared error loss function for the parameter $\lambda$ . | [10]<br>C03 | II,<br>III |
| Q.4(a) State and prove Neymann Pearson Lemma.   | [5]<br>C04  | II         |
| Q.4(b) Suppose $X_1, X_2, \dots, X_n$ is a random sample from $N(\mu, 16)$ . Find the most powerful critical region with $n=16$ and $\alpha = 0.05$ to test hypothesis $H_0 : \mu = 10$ vs $H_1 : \mu = 15$ . Also, find the power of the test.<br>Note: $P(Z < -1.645) = 0.05$   | [5]<br>C04  | III        |
| Q.5(a) State the likelihood ratio test. Prove that for testing simple vs simple hypothesis, Neymann Pearson lemma and likelihood ratio test are equivalent for a given level of significance $\alpha$ .   | [5]<br>C05  | II,<br>IV  |
| Q.5(b) Let $X_1, X_2, \dots, X_n$ is a random sample from $N(\mu, 2)$ . Derive the likelihood ratio test for testing the null hypothesis $H_0 : \mu = 10$ vs $H_1 : \mu \neq 10$ at $\alpha = 0.05$ .<br>( $Z_{\alpha/2} = 1.96$ )  | [5]<br>C05  | III        |

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