BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID-SEMESTER EXAMINATION)

CLASS: IMSc **SEMESTER: V/ADD**
BRANCH: CQEDS **SESSION: MO/2025**

SUBJECT: ED303 - MULTIVARIATE DATA ANALYSIS
TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions, each of 5 marks and a total of 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper, etc., if applicable, will be supplied to the candidates

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Q.1(a)	Consider the symmetric matrix $A = \begin{bmatrix} 2.2 & 0.4 \\ 0.4 & 2.8 \end{bmatrix}$ Find the eigenvalues and the corresponding normalized eigenvectors of A. Also, obtain the spectral decomposition of matrix A.	[3]	1	2
Q.1(b)	Let the $p \times p$ standard deviation diagonal matrix be $V^{1/2} = \text{Diag}(\sqrt{\sigma_{ii}})$, $i = 1, 2, \ldots, p$, and $V^{1/2} \rho V^{1/2} = \Sigma$, ρ and Σ are correlation and variance- covariance matrices respectively. Suppose you have given variance and covariance matrix Σ, can you obtain the correlation matrix using given expression? if yes, write down the expression of ρ matrix.	[2]	1	2
Q.2(a)	The random vector X' = [X1, X2, …, X5] with mean vector $\mu = [2, 4, -1, 3, 0]$ and variance covariance matrix $\Sigma = \begin{bmatrix} 4 & -1 & 1/2 & -1/2 & 0 \\ -1 & 3 & 1 & -1 & 0 \\ 1/2 & 1 & 6 & 1 & -1 \\ -1/2 & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}$, partition X as $X^{(1)} = (X1, X2)$ and $X^{(2)} = (X3, X4, X5)$. Consider $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$ (a) Find the $E(AX^{(1)})$ and $E(BX^{(2)})$. (b) Find the $Cov(A X^{(1)})$ and $Cov(B X^{(2)})$.	[3]	2	3
Q.2(b)	Prove that the sample mean vector is an unbiased estimator of the population mean vector.	[2]	1	3
Q.3(a)	Let X = (X1, X2, X3)`follow a trivariate normal distribution $N3(\mu,\Sigma)$ with mean vector $\mu' = (2, -3, 1)$ and covariance matrix $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ Relabel the variables if necessary, and find a 2×1 vector '*a*' such that X2− *a*′ (X1, X3)′ are independent.	[4]	2	3
Q.3(b)	What do you mean by generalised variance?	[1]	2	2
Q.4(a)	State and prove the Cramer-Wold theorem. Also, explain the importance of this theorem in real-life scenarios.	[3]	1	3
Q.4(b)	If X is distributed as $N_p(\mu, \Sigma)$, what can you say about the distribution of any linear combination of variables, say, a'X, where a is a real vector of size p x 1? Support your answer with a proper argument and obtain the distribution of a'X.	[2]	1	3
Q.5(a)	Let X be a p-dimensional multivariate normal distribution with a mean vector μ and variance-covariance matrix Σ, but $\Sigma = \text{diag}(\sigma_{11}, \sigma_{22}, \ldots, \sigma_{pp})$. Derive the maximum likelihood estimators of μ and Σ.	[4]	2	3
Q.5(b)	Explain the Wishart distribution.	[1]	2	1

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