

BIRLA INSTITUTE OF TECHNOLOGY MESRA, RANCHI  
(MID SEMESTER EXAMINATION MO/2025)

CLASS: IMSc  
BRANCH: QEDS

SEMESTER : I  
SESSION: MO/2025

SUBJECT: ED25101 INTRODUCTORY ANALYSIS

TIME: 02 Hours

FULL MARKS: 25

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates.
  5. All the notations used in the question paper have usual meanings.
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Q.1(a)	Let $A = \mathbb{Q} \cap (0,1)$ . Find closure, derived set of $A$ , and check if $A$ is open.	[2]	CO	BL
Q.1(b)	Find the supremum, infimum, greatest element and least element, if exist, of the set $S = \left\{(-1)^n \left(1 - \frac{1}{n}\right) : n \in \mathbb{N}\right\}$ .	[3]	CO1	2
Q.2(a)	Is the set of all the prime numbers in $[1,10^{10}]$ an open set? Justify your answer.	[2]	CO1	1
Q.2(b)	Using $\epsilon$ –definition show that $\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) = 0$ .	[3]	CO2	2
Q.3(a)	Define Cauchy sequence. Check whether the sequence $\{-1 + (-1)^n\}$ is Cauchy.	[2]	CO2	2
Q.3(b)	Check the convergency of the sequence $\{a_n\}$ where $a_{n+1} = 2 - \frac{1}{a_n}, n \geq 1$ and $a_1 = \frac{3}{2}$ . Hence find the limit $\lim_{n \rightarrow \infty} a_n$ , if exists.	[3]	CO2	2
Q.4(a)	Prove or provide a counter example for the following statement: Every bounded sequence $\{a_n\}$ in $\mathbb{R}$ is convergent.	[2]	CO2	3
Q.4(b)	Using Cauchy's limit theorem evaluate $\lim_{n \rightarrow \infty} \left[ \frac{(4n)!}{((2n)!)^2} \right]^{\frac{1}{n}}$ .	[3]	CO2	3
Q.5(a)	Prove or provide a counter example for the following statement: If the sequence $\{a_n\}$ in $\mathbb{R}$ is convergent then the series $\sum_{n=1}^{\infty} a_n$ is also convergent.	[2]	CO2	3
Q.5(b)	Test the convergency of the series whose $n^{th}$ term is $\{(n^3 + 1)^{\frac{1}{3}} - n\}$ .	[3]	CO2	4

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