BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc **SEMESTER : I**
BRANCH: QEDS **SESSION: MO/2025**

SUBJECT: ED25101 INTRODUCTORY ANALYSIS

TIME: 03 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates.
5. All the notations used in the question paper have usual meanings.

Q.	Question	Marks	CO	BL
Q.1(a)	Define countable set. Is the set $A = \left\{\frac{m}{2^n} : m, n \in \mathbb{Z}\right\}$ countable? Justify your answer.	[4]	CO1	1
Q.1(b)	Let $x_1 = 1$ and $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ for $n \geq 2$. Using monotone theorem, show that the sequence is convergent. Hence find the limit of the sequence.	[4+2]	CO1	2
Q.2(a)	Check the convergency of the following series. $$\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \cdots$$	[5]	CO2	2
Q.2(b)	Using integral test, check the convergency of the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}, p > 0$.	[5]	CO2	2
Q.3(a)	Let $f(x) = \begin{cases} x^2, x < 1 \\ 0, x = 1 \\ 2 - (x-1)^2, x > 1 \end{cases}$. Check the continuity at $x = 1$. If discontinuous, identify the type of discontinuity.	[4]	CO3	3
Q.3(b)	Apply L'Hôpital rule to find the values of a, b and c such that the function $f(x) = \begin{cases} \frac{x(a + b\sin x) - c\,(1 - \cos x)}{x^4}, if\ x \neq 0 \\ 1,\ if\ x = 0. \end{cases}$ is continuous at $x = 0$.	[6]	CO3	3
Q.4(a)	Let $f(x) = \int_0^x t^2 e^{xt}\, dt$. Find $f^4(0)$.	[5]	CO4	4
Q.4(b)	If $y = \cos(m \sin^{-1} x)$, then prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.	[5]	CO4	4
Q.5(a)	Find the asymptote(s) of the following parametric curve, if any. $$x = \frac{3at}{1+t^3};\ y = \frac{3at^2}{1+t^3}, a > 0.$$	[3]	CO5	4
Q.5(b)	Providing all the necessary information, make a rough sketch of the curve $$(x^2 - 1)(y^2 - 4) = 4$$	[7]	CO5	5

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