BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc | **SEMESTER : III**
BRANCH: QEDS | **SESSION : MO/2025**

SUBJECT: ED24207 PROBABILITY II

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

		Marks	CO	BL
Q.1	If the random variables X and Y have the joint density $f(x,y) = \begin{cases} \frac{6}{7}x & for\ 1 \le x+y \le 2, x \ge 0, y \ge 0 \\ 0 & otherwise \end{cases}$ i) What is the joint density of U=2X+3Y and V=4X+Y ? ii) What is the density of $\frac{X}{Y}$?	[10]	C01	II, III
Q.2	Define the Chi-Square distribution. If X_1 and X_2 are two independent Chi-Square variates with n_1 and n_2 d.f respectively, then $\frac{X_1}{X_2}$ is a $\beta_2\left(\frac{n_1}{2},\frac{n_2}{2}\right)$ i.e Beta distribution of second kind.	[10]	C02	II, III
Q.3(a)	Let $Z_1$and Z_2be two independent N(0,1). Define X=$Z_1$and Y= $\rho Z_1+\sqrt{1-\rho^2}Z_2$, $\rho\in(-1,1)$. Find the joint p.d.f of X and Y.	[5]	C03	III
Q.3(b)	State p.d.f of bivariate normal distribution. Let X and Y have joint p.d.f of the form f(x,y) = $k\ exp\left(\frac{-1}{2}\{a_{11}(x-b_1)^2+2a_{12}(x-b_1)(y-b_2)+a_{22}(y-b_2)^2\}\right)$; $\infty < (x,y) < \infty$. *Find k and the correlation coefficient between X and Y.*	[5]	C03	II, III
Q.4	State the conditional expectation of random variable X given Y=y in both discrete and continuous cases. Two random variables X and Y have the following joint probability density function: $f(x,y) = \begin{cases} k(4-x-y)\ ;\ 0 \le x \le 2\ ;\ 0 \le y \le 2 \\ 0 \qquad otherwise \end{cases}$ Find (i) the constant k, (ii) the marginal density functions of X and Y, (iii) conditional density functions , (iv) Var(x), Var(Y) and Cov(X,Y) .	[10]	C04	II, III
Q.5	State and prove Chebyshev's inequality. Let a discrete variate X takes values -1, 0 and 1 with probabilities 1/8 , 6/8 and 1/8. Evaluate P{|X - μ_x | $\ge 2\sigma_x$ } and compare this result with that obtained on using Chebyshev's inequality.	[10]	C05	II, V

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