

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)**

CLASS: B.Sc.
BRANCH: Chemistry

SEMESTER: III
SESSION: MO/2025

SUBJECT: ED107 PROBABILITY I

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
-

		CO	BL
Q.1(a) Suppose a box contains eight black and four white balls. We draw two balls from the box without replacement. Assume that each ball in the box is equally likely to be drawn, what is the probability that both drawn balls are black?	[5]	1	1,3
Q.1(b) Write the definition of conditional probability. Also, write Bayes' formula.	[2+3]	2	1,3
Q.2(a) Write the probability mass function of a Poisson random variable X with parameter $\lambda = 1$. Then, deduce $E[X]$.	[2+3]	5	3
Q.2(b) Suppose X has the following probability mass function: $p(0) = 0.2, p(1) = 0.5, p(2) = 0.3$. Find $Var(X)$.	[5]	3	3
Q.3(a) The joint density function of X and Y is $f(x, y) = \begin{cases} xy, & 0 < x < 1, 0 < y < 2 \\ 0 & otherwise \end{cases}$. Evaluate $P\{X > 1, Y > 1\}$.	[5]	4	2
Q.3(b) Write the definition of moment generating function $\phi(t)$. For independent random variables X and Y , show that $\phi_{X+Y}(t) = \phi_X(t)\phi_Y(t)$.	[2+3]	5	3
Q.4(a) Write the definition of marginal probability density function. The joint density function of X and Y is $f(x, y) = \begin{cases} xy & 0 < x < 1, 0 < y < 2 \\ 0 & otherwise \end{cases}$. Compute $f_X(x)$.	[2+3]	5	1,3
Q.4(b) Write the definition of covariance between two random variables X and Y . Also, prove that if X and Y are independent then $Cov(X, Y) = 0$.	[2+3]	3	3
Q.5(a) The joint density function of X and Y is given by $f(x, y) = \frac{e^{-x/y}e^{-y}}{y}, \quad 0 < x < \infty, 0 < y < \infty.$ Compute conditional probability density function of X , given that $Y = y$. (that is, compute $f_{X Y}(x y)$)	[5]	4, 5	2
Q.5(b) Write the definition of conditional expectation of X , given that $Y = y$. Also, compute $E[X Y = y]$ for the above random variables.	[2+3]	3,5	1, 3