

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)**

CLASS: MTECH
BRANCH: AIML

SUBJECT: CS530 LINEAR ALGEBRA

SEMESTER: I
SESSION: MO/2025

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data handbook/Graph paper etc. to be supplied to the candidates in the examination hall.

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|---|-----|---|----|----|----|----|----|----|----|----|----|---|----|----|----|----|----|----|----|----|----|----|-----|-------------------------------|--|
| Q.1(a) Suppose n features are present in a classification dataset (D) and each feature, $f_i(i=1, \dots, m)$ has k_i discrete values. Find the maximum number instances (records/observations) that D has. Find the minimum and the maximum number of instances in D if each of the features has at least 3 and at most r discrete values. | [5] | Apply
[CO1] | | | | | | | | | | | | | | | | | | | | | | | |
| Q.1(b) How does the concept of dot product of vectors help in data science? Explain with example. | [5] | Apply
[CO2] | | | | | | | | | | | | | | | | | | | | | | | |
| Q.2(a) Find the value of m such that the vector is a linear combination of vectors: and gives zero (i.e., the vectors are linearly dependent). How will you find the dimension of a vector space? What is the importance of finding dimension of vector space? | [5] | Understand
[CO2] | | | | | | | | | | | | | | | | | | | | | | | |
| Q.2(b) Differentiate between direct and iterative approaches to solve a set of linear equations. Solve the set of linear equations: $2x+y+z=10$; $3x+2y+3z=18$; $x+4y+9z=16$, using Gauss Elimination technique. | [5] | [Understand,
Apply]
[CO2] | | | | | | | | | | | | | | | | | | | | | | | |
| Q.3(a) For which type of problems do you prefer to apply the concept of eigen equation? Give some applications of the Eigenvalues and eigenvectors of a matrix. | [5] | Understand
[CO1] | | | | | | | | | | | | | | | | | | | | | | | |
| Q.3(b) Find the eigen values and the corresponding eigen vectors of the matrix:
$\begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$ Suppose, a dataset (D) has 4 attributes: A_1, A_2, A_3 and A_4 , and the tuple-values of an eigen vector v_1 (for $\lambda = \lambda_1$) for covariance matrix of D are (0.36138659 -0.08452251 0.85667061 0.35828920). How will you find the values of principal component (PC) for this v_1 ? | [5] | Analyse
[CO4] | | | | | | | | | | | | | | | | | | | | | | | |
| Q.4(a) Let X and Y be two discrete random variables.
<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr> <td>X</td><td>65</td><td>72</td><td>68</td><td>60</td><td>70</td><td>63</td><td>69</td><td>66</td><td>71</td><td>67</td> </tr> <tr> <td>Y</td><td>78</td><td>85</td><td>62</td><td>91</td><td>75</td><td>88</td><td>70</td><td>95</td><td>68</td><td>80</td> </tr> </table> | X | 65 | 72 | 68 | 60 | 70 | 63 | 69 | 66 | 71 | 67 | Y | 78 | 85 | 62 | 91 | 75 | 88 | 70 | 95 | 68 | 80 | [5] | Understand,
Apply
[CO3] | |
| X | 65 | 72 | 68 | 60 | 70 | 63 | 69 | 66 | 71 | 67 | | | | | | | | | | | | | | | |
| Y | 78 | 85 | 62 | 91 | 75 | 88 | 70 | 95 | 68 | 80 | | | | | | | | | | | | | | | |
| Q.4(b) Find the Cov(X, Y) and then infer relationship between these two variables. Establish the correlation coefficient relationship (r_{XY}) between two random variables: (X and Y) as: $-1 \leq r_{XY} \leq 1$. | [5] | Understand
[CO3] | | | | | | | | | | | | | | | | | | | | | | | |
| Q.5(a) Let $n=p \times q$, where p and q are two prime numbers. How the public and the private keys are generated as per RSA algorithm? What information may be publicly shared? What information do receiver know? Why p and q are chosen very large? | [5] | [Remember,
Understand,
Analyse]
[CO4] | | | | | | | | | | | | | | | | | | | | | | | |
| Q.5(b) Let us consider two tests (T_1 and T_2) on a subject conducted in two consecutive semesters. The following information are given below for two samples (S_1 and S_2) taken from T_1 and T_2 . In S_1 , mean and standard deviation values are 80 and 5 respectively, and person: A's score in T_1 is 87. In S_2 , mean and standard deviation values are 70 and 6 respectively and person: B's score in T_2 is 77. Compare between two persons, using Z-statistics. Who is the better performer why? | [5] | [Understand,
Apply]
[CO5] | | | | | | | | | | | | | | | | | | | | | | | |