BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BTECH **SEMESTER : III**
BRANCH: CSE **SESSION : MO/2025**

SUBJECT: CS24203 MATHEMATICS FOR COMPUTER SCIENCE

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	**BL**
Q.1(a)	For each of the following pair of functions f(n) and g(n), find an appropriate positive constant c such that f(n)<= cg(n) for all n >1. a. $f(n)=n \sqrt{n} + n^2$ and $g(n) = n^2$, b. $f(n)=n^2 - n + 1$, $g(n)= n^2/2$.	[5]	1	3
Q.1(b)	A digital system has four input variables: A, B C D . The output F is high for the following min terms: F(A,B,C,D) = Σ m(0,2,4,5,6,7,10,13,14) i. Construct a 4-variable Karnaugh Map for the function F. ii. Identify and circle all prime implicants and essential prime implicants. iii. Derive the simplified Boolean expression for using the Karnaugh Map. iv Implement the simplified expression using basic logic gates (AND, OR, NOT).	[5]	1	3
Q.2(a)	A sensor network monitors the number of packets drops per minute on a critical link. Let X be the discrete random variable denoting the number of drops in a one-minute interval. Past measurements suggest X has support {0,1,2,3} with unknown probabilities p0, p1, p2, p3. The network engineer models the probability mass functions(pmf) with the following constraints: • P(X = 0) = p0. • P(X = 1) = α p0. • P(X = 2) = β p0. • P(X = 3) = 1 − p0(1 + α + β). For a particular link an engineer estimates α = 2 and β = 1.5. Answer the following parts. a. Compute p0, p1, p2, p3 explicitly for α = 2 and β = 1.5. Verify the pmf sums to 1 and state any constraints required for validity. b. Compute the expectation E[X] and variance Var(X). Show each step (write sums explicitly). Compute P(X ≥ 2).	[5]	2	2
Q.2(b)	A communication system transmits signals over a noisy channel. The received signal amplitude (X) is modelled as a continuous random variable with probability density function (PDF): $F_x(x) = \mu e^{-\mu x}$ x>=0 = 0 x<0 Where the μ > 0 is a parameter related to the noise characteristics. Answer the following questions (i) Verify that $F_x(x)$ is a valid probability density function. (ii) Find Expectation & Variance, Derive the mean E[X] and variance Var(X) (iii) Suppose the system fails if the received amplitude (X) falls below a threshold (T). At this situation derive the probability of system failure (P(X < T)) and for μ= 0.5) and (T = 3), compute this probability numerically. (iv) Discuss how increasing μ (representing higher noise intensity) affects system reliability.	[5]	2	3

PTO

Q.3(a)	When a algebraic structure is called a group? Write all its properties. What is subgroup and cyclic subgroup give example of each and justify them.	[5]	3	1
Q.3(b)	In an encryption system, a cyclic multiplicative group is constructed using the set $G = \{1,2,3,4,5,6,7,8,9,10\}$ under multiplication modulo 11. A message is encoded using the key transformation $C \equiv M^k (\text{mod} 11)$ where M is the original message (as an element of G), k is a secret key, and C is the encrypted ciphertext. i. Prove that (G, X mod11) forms a group. Discuss each group property explicitly. ii. Find the multiplicative inverse of **7** modulo 11 and show its use in *decrypting* the message. Iii If the encryption key k = 3 and the original message is M = 7,compute the ciphertext	[5]	3	3,6
Q.4(a)	Define the vector space and linear independency and dependency of vectors. Discuss with examples	[5]	4	1
Q.4(b)	Find the eigenvalues and eigenvectors of the given matrices: $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ and $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$	[5]	4	2,4
Q.5(a)	Explain the following Chebyshev's Inequality, Selberg's Inequality, Jensen's Inequality, Cauchy–Schwarz Inequality, Kraft's Inequality with example.	[5]	5	1
Q.5(b)	Define the Lagrange multiplier and solve the following: Find the maximum value of the function f(x, y) subject to the constraint g(x, y). • Maximize: $f(x,y)= x +2y$ • $g(x,y): x^2+y^2=4$ $X, y \geq 0$	[5]	5	1,4

:::::21/11/2025:::::E