BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER: V**
BRANCH: Maths and Computing **SESSION: MO/2025**

SUBJECT: CS241 DESIGN AND ANALYSIS OF ALGORITHMS
TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. **The question paper contains 5 questions each of 5 marks and total 25 marks.**
2. **Attempt all questions.**
3. **The missing data, if any, may be assumed suitably.**
4. **Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

			CO	BL
Q.1(a)	Given $f(n) = (n + a)^b$, $for\ any\ real\ constants\ a\ and\ b, show\ that\ f(n) = \theta(n^b)$	[2]	1	2
Q.1(b)	Prove or disprove the following statements: I. Is $2^{10n} = O(2^n)$ II. $f(n) + g(n) = \Theta(\max\{f(n), g(n)\})$, where f(n) and g(n) are positive asymptotic functions.	[3]	1	2
Q.2(a)	Solve the following recurrence: $T(n) = 3T\left(\frac{n}{2}\right) + n$	[2]	1	3
Q.2(b)	Given two integers a=2468 and b=1357, find the product $a \times b$ using the **Karatsuba algorithm** for integer multiplication.	[3]	1	1
Q.3(a)	What are the minimum and maximum numbers of elements in a heap of height h?	[2]	2	1
Q.3(b)	Show that with the array representation for storing an n-element heap, the leaves are the nodes indexed by $\lfloor n/2 \rfloor + 1,\ \lfloor n/2 \rfloor + 2,\ \dots,\ n$. Illustrate the operation of HEAP-EXTRACT-MAX on the max-heap A=⟨22, 17, 15, 10, 9, 8, 12, 3, 5, 6, 4⟩.	[3]	2	2
Q.4(a)	Show that the running time of QUICKSORT is $\theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.	[2]	2	2
Q.4(b)	Suppose $X = \{x_1, x_{2,} \dots x_n\}$ is a sorted array of n integers and x is an integer. Design an efficient algorithm to determine whether there are two elements in X whose sum is exactly x. Also, find out the complexity of the algorithm you designed.	[3]	2	2
Q.5(a)	Given n elements, the second maximum can be found trivially with 2n − 3 comparisons in the worst case: n − 1 comparisons for the first maximum and n − 2 comparisons for the second maximum. Try to find out an efficient method (that takes fewer than 2n−3 comparisons) to find the second maximum. Note that here we are not interested in the asymptotic complexity but the exact count in the worst case.	[2]	2	3
Q.5(b)	Given a set of intervals with their (start time, finish time, weight) as S= {(2,5,6), (1,4,3), (6,8,5), (5,7,4), (3,9,7), (8,10,2), (2,6,8)}. Find a set of non-conflicting intervals with maximum total weight.	[3]	3	2