BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

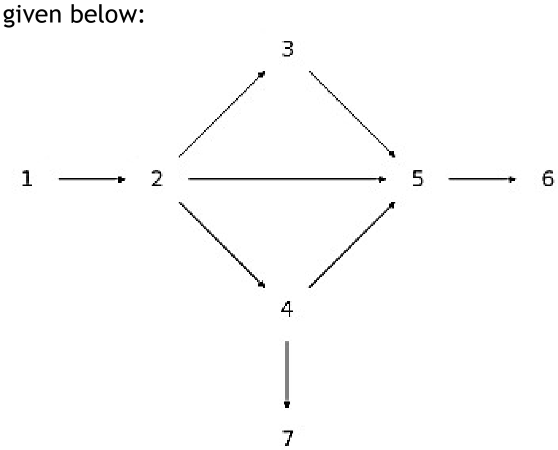
CLASS: IMSC **SEMESTER : V**
BRANCH: MATHS AND COMPUTING **SESSION : MO/2025**

SUBJECT: CS241 DESIGN AND ANALYSIS OF ALGORITHMS

TIME: 3 HOURS **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Prove or disprove the following statements: i. Suppose there exist two asymptotically positive functions f(n) and g(n) such that $f(n) = O(g(n)). Then, 2^f(n) = O(2^{(g(n))})$ ii. Suppose that f and g are two functions (taking nonnegative values) such that $g = O(f)$. Then $f + g = \Theta(f)$. (2.5+2.5)	[5]	1	2
Q.1(b)	Solve the following recurrence using the recursion tree method: $T(n) = T(n/2) + cn.$ Note that if you can find an upper bound for T(n), it will be ok. Also, solve the following recurrence using any method: , $T(n) = 2T(n-1), T(0) = 1$ (2.5+2.5)	[5]	1	2
Q.2(a)	Convert the given array A = <27, 17,3,16,13,10,1,5,7,12,4,8,9,0> into a Max-heap. In a binary max-heap containing 100 elements, if the root is at position one, find the position of the parent of a node i that is present at position 19 and check if i is a left or right child of its parent. (3+1+1)	[5]	2	3
Q.2(b)	Let $X = x_1, x_2, \ldots x_n$ *be* an array of n distinct integers. Design an efficient algorithm to determine whether there are three elements in X whose sum is exactly 0. Find the time complexity of the algorithm you designed. Write a pseudocode to sort an array of n numbers in O(n) time, where all the elements are either 0 or 1. (3+2)	[5]	2	2
Q.3(a)	Given an undirected graph G = (V, E), design and analyse an efficient algorithm to determine if G contains a cycle of odd length. A cycle of odd length is referred to as an odd cycle. (Hint: Check whether the graph is bipartite or not). (2) Consider the figure given below:	[5]	3	3



Classify the edges of the graph after performing DFS traversal and show that it is a DAG. (3)

Q.3(b)	Consider the subset sum problem. Find the recurrence relation for the problem and its base cases. Use your recurrence to determine whether there exists a subset of S = {3, 4, 7, 10, 12} with target sum T = 17. If yes, then find the subset. (2+1.5+1.5)	[5]	3	2

PTO

Q.4(a)	i. Consider the interval scheduling problem. Suppose in order to solve the problem, the greedy choice selected is "Choose the interval first that has the smallest duration, i.e., the interval which has the least (finish time- starting time). Discuss whether this strategy will give you the optimal solution or not. (1.5) ii. Suppose that some edge costs in a directed graph G are negative. Ensure all edge costs are positive by adding a fixed, positive bias to each cost. Then run Dijkstra's algorithm on this updated graph. Give an example to demonstrate that this algorithm may fail to give the shortest paths in the original algorithm. (2) iii. Kruskal's algorithm works according to a greedy strategy. In other words, there exists a greedy strategy to solve the MST problem using a greedy algorithm. Justify. (1.5)	[5]	4	3
Q.4(b)	Consider the following set of symbols and their frequencies representing characters in a text file:	[5]	4	3

Symbol	Frequency
A	5
B	9
C	12
D	13
E	16
F	45

i. Compute the fixed-length code for each symbol, assuming that all symbols are encoded with the same number of bits. (1)

ii. Using Huffman coding, construct the optimal prefix code and determine:

- The Huffman code for each symbol. (2)
- The average code length. (1)
- The total number of bits required to encode the file using Huffman coding. (1)

Q.5(a)	Write the decision version of the Vertex Cover Problem. Show that the vertex cover problem is in NP. Define an NP Complete Problem. Suppose you know that the decision version of the independent problem is NP Complete. Using this information, can you demonstrate that the vertex cover problem is also NP-complete? (1+1+1+2)	[5]	5	2
Q.5(b)	State the decision version problem of SAT and 3-SAT. Consider the following Boolean formula in Conjunctive Normal Form (CNF): $\varphi = (x_1 \vee x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee x_3) \wedge (x_2 \vee x_5 \vee x_6 \vee \neg x_7 \vee x_8)$. Construct an equivalent 3-CNF formula. (2+3)	[5]	5	2

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