

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(END SEMESTER EXAMINATION)**

**CLASS: BTECH  
BRANCH: AI AND ML**

**SEMESTER : III  
SESSION : MO/2025**

**SUBJECT: AI24201 MATHEMATICS FOR AI & ML**

**TIME: 3 Hours**

**FULL MARKS: 50**

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Before attempting the question paper, be sure that you have got the correct question paper.
  5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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- Q.1(a) (i) Create a relationship for given data points (-5, 27), (-4, 18), (-3, 11), (-2, 6), (-1, 3), (0, 2), (1, 3), (2, 6), (3, 11), (4, 18), (5, 27) [2+1+2] CO BL  
2,5 5
- (ii) Comment kind relation (e.g. one to one, one to many, many to one and many to many) it is.
- (iii) Comment whether your created relationship is a function or not. Justify your answer.
- Q.1(b) A streaming platform wants to build a movie similarity graph to support its recommendation system. [1+2+1+1] 2,5 4
- You are given a list of movies and their pairwise similarity scores (on a scale of 0 to 1), based on user viewing history and genre matches.

Movie Pair	Similarity Score
Movie A - Movie B	0.8
Movie A - Movie C	0.6
Movie B - Movie D	0.7
Movie C - Movie D	0.9

- (i) Draw a labelled graph above situation.
- (ii) Find whether your graph is a complete graph or having cycle or both. If yes give reason of cycle present in graph and vertices present in cycle.
- (iii) Create an adjacency list for your graph.
- (iv) Find most similar movies in your graph, write your answer with justification.
- Q.2(a) (i) Find the mean, median, and mode of 13, 16, 12, 14, 19, 12, 14, 13, 14. [3+2] 2,5 3
- (ii) The Ted Talks Analysis System employs machine learning algorithms to analyze user preferences and suggest relevant talks that align with individual interests. Identify the problem whether it is supervised or unsupervised. Justify your answer.
- Q.2(b) Consider the given Dataset. Apply Naive Bayes' Algorithm and Predict that if a fruit has the following properties then which type of the fruit it is [1+2.5+1.5] 4 3
- Fruit = {Yellow, Sweet, Long}
- Frequency Table:

Fruit	Yellow	Sweet	Long	Total
Mango	350	450	0	650
Banana	400	300	350	400
Others	50	100	50	150
Total	800	850	400	1200

- (i) Calculate Prior probabilities?
- (ii) Calculate Conditional probabilities?
- (iii) Calculate Posterior probabilities? And find fruit?

**PTO**

Q.3(a) (i) Identify vector/scalar [2+3] 1 3  
 (i) Velocity (ii) Time (iii) Displacement (iv) Speed

(ii) Let's consider two vectors in the vector space  $V=R^2$  over the field of real numbers  $K = R$ . Find a and b if they are linearly independent.

$v_1 = (3 \ 1)^T$   
 $v_2 = (2 \ 5)^T$

Q.3(b) [2+3] 3 2

$A = \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix}$

(i) Find singular values of given matrix A =  
 (ii) Proof that  $A^T A = A A^T$

Q.4(a) (i) Compare of PCA and LDA in terms of objective function, interpretability and approach. [2+3] 2 2

(ii) A financial analyst is studying the relationship between hours of study and exam scores for a group of students to determine if there is a positive association between them. The following data was collected from a sample of 5 students:

Hours of Study (X)	Exam Scores (Y)
2	60
4	65
6	70
8	75
10	80

Calculate the sample covariance between hours of study (X) and exam scores (Y).

Q.4(b) (i) Find differentiation of given matrix [2+3] 2,4 3

$$\begin{bmatrix} 6x^2 & \cos x & 5 \tan x \\ 17 & 90x & \sin x + e^{16x} \\ 3 \sin 4x + 32x^2 & \ln 16x & e^{\cos x + 4x} \end{bmatrix}$$

(ii) Use the Lagrange interpolation method to estimate the value of P(x) at x = 4 for the following data points: (1, 2), (2, 4), (3, 6), and (5, 10)

Q.5(a) (i) Write linear regression algorithm using gradient descent. [2+1+2] 4 5  
 (ii) Identify appropriate kernel for the relationship between features and the target variable is non-linear. Justify your answer.

(iii) Identify overfitting, Under fitting and Balanced model using given metric

Metric	Model A	Model B	Model C
Train Accuracy	0.75	0.99	0.93
Test Accuracy	0.72	0.80 → 0.68	0.90
Precision (Test)	0.70	0.95	0.91
Recall (Test)	0.65	0.60	0.88
F1-Score (Test)	0.67	0.73	0.89
AUC-ROC (Test)	0.70	0.76	0.93

Q.5(b) Calculate accuracy, precision, recall, F1-score, AUC-ROC for given confusion matrix. [5]

3 4

n=165	<b>Predicted: NO</b>	<b>Predicted: YES</b>	
<b>Actual: NO</b>	TN = 50	FP = 10	60
<b>Actual: YES</b>	FN = 5	TP = 100	105
	55	110	

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